Price Categorization, Limited Memory, and Competition

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ABSTRACT

This paper investigates the effects of a limited consumer memory on the price competition between firms. It studies a specific aspect of memory, namely, the categorization of available price information that the consumer may need to recall for decision making. The paper analyzes competition between firms in a market with uninformed consumers who do not compare prices, informed consumers who compare prices but with limited memory as well as informed consumers who have perfect memory. We show that heterogeneity in the memory capacity of the informed consumers who compare prices softens price competition and allows firms to achieve higher profits. To focus on the effect of memory limitations we examine the limit market in which all the informed consumers who compare prices have limited memory and confirm the robustness of our main results across different categorization processes and market conditions. We find that the optimal partition for the consumers calls for finer categorization towards the bottom of the price distribution. Thus consumers have a motivation to invest in greater memory resources in encoding lower prices in order to induce firms to charge more favorable prices. The interaction between the optimal categorization strategies of the consumers and the price competition between the firms is such small initial improvements in recall move the market outcomes quickly to the case of perfect recall. Thus even with few memory categories the expected price consumers pay and their surplus is close to the case of perfect recall. There is thus a suggestion in this model that market competition adjusts to the memory limitations of consumers.

1. Introduction

In the literature on imperfect price competition an implicit assumption is that consumers who compare prices across firms are able to perfectly recall the prices encountered and to use them in decision-making. However, there is a substantial body of psychological research that examines the effect of memory limitations on consumer choice among available alternatives.[1] Limitations on short-term memory mean that consumers would not be able to perfectly recall relevant price information and consumers are more likely to face greater short-term memory constraints in environments with higher levels of information. Such limitations would be particularly significant in complex product markets where consumer comparisons across firms are based on the recall of not just the numerical posted prices, but on the full price that includes other associated monetary aspects such as payment terms, financing, delivery, warranty offers or other optional features. In such environments with significant information processing requirements consumers make decisions using heuristics that help them to form suitable price impressions.

An important heuristic to deal with the abundance of information is grouping events, objects or numbers into categories on the basis of perceived similarities (see Rosch and Mervis 1975). Accordingly, memory limitations in our model pertain to the categorization scheme that consumers use to encode and recall prices. The purpose of this paper is then to highlight the effect of a specific aspect

[1] See Alba, Hutchinson and Lynch (1991) for a survey of the information processing literature that examines the effect of memory on consumer choice.

of limited consumer memory on the competitive pricing strategies of firms: consumers are able to recall prices only as categories. We develop a model of market competition between firms who face consumers who compare prices, but have the ability to recall previously encountered market prices only as categories.

The model of this paper highlights the following aspects of memory limitations: First, we model memory limitations as the ability to recall only the category of a previously observed price rather than the actual price realization. Second, we take the analysis a step further by embedding consumers with limited memory in a market setting involving price competition between firms. In equilibrium, the pricing strategies of the firms take into account the memory limitations of consumers. Finally, following Dow (1991) we also consider the problem of consumers who have limited memory, but who are aware of this limitation and take it into account to make the best possible decision. In other words, we explicitly consider the "encoding" decision of consumers given the information environment.[2] In doing so, we provide a framework that investigates how memory limitations is affected by, and in turn, governs market equilibria.

Specifically, we model duopoly price competition between firms in a heterogeneous consumer market with memory limitations. Each firm has a group of uninformed consumers that would consider shopping only at that firm as long as the price is below a common reservation price. Because these consumers consider only their favorite firm, they do not have the need to remember prices and therefore memory limitations do not matter for this group. Another group of informed consumers have perfect memory and buy at the firm offering the lowest price. The final group of consumers are informed consumers but with limited memory. These consumers compare prices before buying, but have the ability to recall only the category of a previously encountered price. We consider the effects across two distinct types of categorization processes to represent memory limitations. First, we consider the categorization process similar to Dow (1991) in which consumers compare the actual price realization at one firm to the recalled category of the price at the other firm. A consumer with limited memory contacts both firms in sequence and compares their prices. The consumer cannot recall the exact price that was observed in the first firm, instead the consumer divides the price distribution into several categories and can recall only the category that the actual price realization fell into. The consumer observes the actual price charged at the second firm and decides which firm to buy from based upon the actual price realization and the expected price at the first firm conditional on the recalled category. This decision process provides a parsimonious framework to analyze the value of remembering price information from competing sellers in a market.[3] Next, we consider the effects of an alternative type of categorization process in which the limited memory consumers

[2]Thus the model is in the spirit of bounded rationality as defined in Simon (1987) where the decision maker makes a rational choice that takes into account the cognitive limitation of the decision maker.

[3]As in Dow (1991), this process is not to be interpreted as one in which consumers search across the firms. Nevertheless, in this paper, we also consider the case in which the decision process of the limited memory consumers involves the initial decision of whether to compare prices at all, after observing the price at the first firm.

decide by comparing the category labels of the prices of the two firms. This would represent the a situation in which consumers can only compare the impressions of the prices from both firms.

We find the Nash equilibrium of firms' pricing strategies and the surplus maximizing categorization strategy for limited memory consumers. We show that consumer heterogeneity due to the differences in the memory capacity of the consumers softens price competition between the firms. Thus in a market with both limited memory and perfect memory consumers (for a given number of uninformed consumers) the equilibrium profits of the firms are higher than in the standard model in which all the informed consumers have perfect memory. Further, the expected profits of the firms always increase when there are a greater number of uninformed consumers. To focus on the effect of recall imperfections we examine the limit market in which all the informed consumers who compare prices have limited memory. The equilibrium in such a market consists of firms choosing a mixed strategy with a support that is a union of finite number of prices. Firms adjust their pricing strategies and charge exactly the same number of prices (with positive probability) as the number of categorizations that consumers have in their memory. This implies that the strategic behavior of firms adjusts to limited memory. There is therefore a suggestion in this model that market competition adjusts for the imperfect recall of consumers.

Our analysis establishes several results about the effect of limited memory that are remarkably robust across the different types of categorization processes and market conditions. We find that the optimal partition for the consumers calls for finer categorization towards the bottom of the price distribution. This implies that consumers devote memory resources in encoding lower prices in order to induce firms to charge more favorable prices. The interaction between the optimal categorization strategies of the consumers and the price competition between the firms is such small initial improvements in recall move the market outcomes quickly to the case of perfect recall. Thus even with few memory categories the expected price consumers pay and their surplus is close to the case of perfect recall.

We also investigate the robustness of our results to different representations of the category. Specifically, in addition to the mean, we consider the cases when consumes use category representations such as the median as well as exogenous rules such as the top or the bottom of the category. The key results of the paper are robust to different endogenous representations of the category such as the median (rather than the mean), as well as exogenous representations such as the top or the bottom of the category. Interestingly, when consumers use the mean of the category which is endogenous to the equilibrium firm actions, their surplus under optimal categorization strategy is actually higher than when consumers use exogenous rules such as the bottom or the top of the category. Thus if consumers in a market were to learn through experience over time to do the best for themselves and were motivated to make the best possible purchase decisions, our analysis suggests that it is optimal for them to recall the category mean price rather than to use an exogenous rule.

1.1. Related Research

A useful way of modeling bounded rationality in the literature has been to enforce limitations on the information processing of the decision maker. The decision maker cannot perfectly convert the inputs she receives to the optimal outputs she needs to choose. Our paper can be seen as related to a class of such models that involve the specific modeling of partitions or categories. The consumer receives as an input a signal (e.g., prices) that she cannot perfectly recall. The consumer partitions the entire set of potential signals and classifies the one received into a partition. The action taken by the consumer is identical for all signals that fall within a category. In the literature, exogenously given partitions are the most common. Along these lines there is research in game theory that models the use of finite automata mechanisms in repeated games (see Kalai 1991). In contrast to the literature on exogenous partitions, Dow (1991) investigates the question of the optimal choice of the partitions by consumers. However, in Dow consumers face exogenously fixed price distributions. In our paper, the price distributions are a result of the competitive market equilibrium. In addition to examine the case that the partitions are exogenously given, we are also interested in the optimal choice of the partition structure by consumers who face price distributions offered by firms in a market equilibrium. Rubinstein (1993) analyzes a model in which a firm price discriminates between consumers who have different memory capacities by choosing a random lottery of prices which consumers categorize. In that paper the prices are a result of the choice of a monopoly firm, rather than the result of competition between firms.

Our paper is also related to the recent literature on the effect of bounded rationality on market interaction. For example, Spiegler (2006) develops a market model in which firms are fully rational as in the present paper, but where consumers are limited in their ability to process information about firm characteristics and form boundedly rational expectations about the firm. The paper shows that competitive firms in such markets choose actions that obfuscate consumers leading to a loss of consumer welfare. Camerer, Ho and Chong (2004) is an approach to modeling bounded rationality in the form of cognitive limitations of players that lead to lack of rational expectations of the beliefs that agents have about other agents. In addition, this paper is related to the recent research direction which examines how common biases in consumer decision making affect strategic interaction. For example, Amaldoss et. al (2008) show that the psychological bias called the asymmetric dominance effect may facilitate coordination in games. Another recent example of psychological biases in strategic decision making is Lim and Ho (2007) who study the effect of counterfactual thinking related payoffs when retailers in a channel are faced with multi-block tariffs.

Our paper also adds to a literature that models the effects of memory on economic decisions. Mullainathan (2002) models some specific psychological aspects of human memory for economic behavior, namely, rehearsal and associativeness and in doing so provides a structure to understand when individuals will under or overreact to news. Mehta, Rajiv and Srinivasan (2004) develop a structural econometric model to analyze the role of imperfect recall of consumers for choice decisions of frequently purchased products. Their aim is to characterize the role of forgetting over time on the brand choice decisions of consumers. Similarly, another recent paper Shapiro (2006) presents a model of how advertising changes the consumers' recollections of past experiences with products. Our paper adds to this literature by focusing on the specific memory phenomenon of categorization and examines how this affects the competition between firms.

The rest of the paper is organized as follows: Section 2 presents the model which is analyzed in Section 3 which also presents the results of the paper. In Section 4 we discuss the robustness of our results across different consumer decision processes and categorization schemes. Finally, Section 5 concludes and provides a summary.

2. The Model

Consider two symmetric competing firms indexed by j (where $j = 1, 2$) selling a homogenous product. Let the marginal cost of production of each firm be constant and assume it is equal to zero. The market is comprised of a unit mass of consumers with each consumer requiring at most one unit of the product. Consumers have a common reservation price which is normalized to one without loss of generality. The market consists of the following group of consumers: A group of *uninformed* consumers of size 2γ who randomly purchase from either firm with equal probability as long as the price is below their reservation price.[4] These consumers do not compare prices across the firms and therefore there is no role for memory in facilitating price comparisons for these consumers. Another group of fully *informed* consumers with perfect memory for market prices of size 2α who can costlessly compare the prices offered by both the firms. Finally, in order to examine the effect of memory limitations we have a group of informed consumers with limited memory of size $2\beta = 1 - 2\gamma - 2\alpha$. These consumers also compare prices offered by the two firms. But when they compare the two prices, one price is recalled from an imperfect memory of a previously encountered price. Therefore, the recalled price may be different from the actual price charged by the firm. Consumers have memory limitations but they are aware of that fact. In other words, they realize that the recalled price might be inexact and act optimally given the memory constraints. For model tractability, we assume that consumers in the 2β segment is homogenous, i.e., they have the same memory structure and adopt the same categorization rules.

We model the decision process of these limited memory consumers in a simple framework designed to highlight the value of remembering information. First, we consider a three-stage decision process

[4] Alternatively, γ of these consumers can also be assumed to consider purchasing only from one of the two firms, while remaining γ consider the other firm.

that is similar to Dow (1991) and in which the limited memory consumers who contact both the firms as follows: In the first stage, they observe the price at a firm and encode this price (half of the consumers observe Firm 1 while the other half observe Firm 2). In the second stage, the consumers observe the exact price at the other firm, compare it with the price recalled from their memory, and decide whether or not to buy the product at the current firm. If the actual price observed is lower than or equal to the price recalled from memory, the consumer will purchase one unit of the good at the second firm (provided that the price is not higher than the reservation price).[5] Finally, in the third stage, if the consumers did not purchase at the second stage from the second firm they purchase from the original firm provided that the price there is below the reservation price.[6] Note that this decision process is designed to represent price comparisons between the prices of the two firms, but when the price of one of the firms can only be recalled imperfectly by the consumers.

2.1. Interpretation of the Decision Process

We now discuss some issues regarding the interpretation of the decision process. First, one might ask why remembering a price is at all imperfect. However, in choosing to buy consumers do not always use information that simply comprises of a posted price number. Often the price might be associated with numerous other details such as service fees, payments schedules, warranty characteristics etc. When consumers compare across firms it is this "impression" of price consisting of several other details in addition to the numerical posted price that might be relevant making price comparisons imperfect. Second, the reader can ask why the consumer might not simply follow the strategy of noting down the price rather than (imperfectly) recalling it. Once again, the interpretation that must be held is that what is relevant is not only the numerical posted price, but the full price impression which includes all the other details and specifications that are relevant for comparison across the firms. Thus the model is most relevant for complex purchasing situations where the full price facing the consumer consists of not only the quoted price but also various types of discounts, trade-in payment, financing offers, warranty, delivery schedule assembly charges etc. This makes the posted numerical price just one of the inputs to decision making.[7]

[5]Note that we assume that if the observed price at the second firm is exactly identical to the price that is recalled from memory, the consumer will buy at the second firm.

[6]This decision process can also have other interpretations pertaining to the broader class of problems of communication constraints between agents in organizational settings. For example, it can be seen as reflecting communication constraints when the decision making team consists of a team of two agents. Agent 1 observes the price in the first firm and sends a message to agent 2, who then has the discretion to decide after observing the price at firm 2. Here the constraint can be interpreted as a limit on the set of words/messages that can be sent.

[7]Indeed, the possibility of consumers being able to costlessly note down the price provides a possible justification for the consideration of perfect memory and limited memory consumer segments. The perfect memory consumers can be seen as those consumers who can costlessly and perfectly note down all the relevant "full price" information that is necessary for across firm comparisons, while the limited memory consumers find it costly (or are unable) to note down the relevant information and therefore rely on their recall of the information.

2.2. Modeling the Decision Process

The details of the model of consumers' categorization of price information are as follows: Consumers impression of the price of an item in a store is often summarized by the phrase "this good is expensive" or "it is inexpensive." Specifically, suppose that the consumers in the limited memory group use $n+1$ memory categories to encode the price information. They divide the entire range of possible prices into $n+1$ mutually exclusive and exhaustive categorizations or partitions. When they observe a price they classify it to one (and only one) category.

The categorization process can be understood as follows: When consumers deal with firms in a market they would have encountered many prices (and the total "price" faced by consumers may have several aspects). This makes price comparisons imperfect and the categorization heuristic captures this imperfection. Given this we appeal to the literature in cognitive psychology which shows that despite the fact that individuals have limited short term memory, the mind is sophisticated in constructing heuristics to react to the limitations in information processing. Indeed, in the specific context of the categorization heuristic Rosch (1978) provides the best support when she argues that individuals aim for "cognitive efficiency" by minimizing the variation with each category. Further, the findings of the more recent experimental literature on the automaticity of categorical thinking is also consistent with our model in that subjects may develop and use complex categorization rules without even being consciously aware of computing it (see Bargh 1994, 1997), while at the same time their ability to recall information from short-term memory might be limited. This characterization in the literature is consistent with our model and price categorization is akin to a long-term heuristic that may be automatically formed by the mind of the consumer due to experience over time with the market prices, which then helps the consumer to make optimal decisions during a specific purchase occasion.

Let the set of categories be $\{C_i\}_{i=1}^{n+1}$ and assume without loss of generality that they are indexed in increasing order of prices ($j > k$ implies that $p > q$ for every $p \in C_j, q \in C_k$). Denote the set of cutoff prices as $\{k_i\}_{i=1}^{n}$ where k_i, separates category i from category $i+1$. From our previous assumption on the order of the categories we have that $k_1 < k_2 < ... < k_n$. Because the categories are exhaustive and mutually exclusive, k_i belongs to one and only one of them, we assume that each category is a set that is open to the left which implies $k_i \in C_i$ for $i = 1, \cdots, n$. Therefore, $k_{i-1} < p \leq k_i \implies p \in C_i$. Finally, for the sake of completeness we define k_{n+1} and k_0 to be the highest and lowest possible prices charged by a firm respectively (clearly, $k_{n+1} = 1$). Therefore, the category C_i is defined as the set of prices $(k_{i-1}, k_i]$ for $i = 1, 2, \cdots, n+1$. Suppose $n = 1$, then one might think of consumers partitioning observed prices into an "inexpensive" or an "expensive" category.

Consumers remember the expected price charged in a category as the representative of that category. Let $\{m_{ij}\}_{i=1}^{n+1}$ be the set of the mean of the prices that each firm j charges in each category, then $m_{ij} = E\left[p_j \mid p_j \in C_i\right]$. Clearly $k_{(n+1)} \geq m_{(n+1)j} > k_n \geq m_{nj} > \cdots > k_1 \geq m_{1j} > k_0$. Figure 1 shows the categorization scheme with the categories, cutoffs, and mean prices.

To summarize, a limited memory consumer observes a price in the first firm classifies it into a category. When she compares it with the actual price at the other firm, she only recalls which category the first price fell into. The consumer uses the mean price of the recalled category for price comparison with the actual price observed in the second firm. The parameter n captures the precision of consumers' price recall or the degree of memory. If $n = 0$, consumers can only recall the mean price charged, thus representing the case of "no memory." If $n = 1$, consumers divide the firms' prices into two categories: high prices or low prices, and recall only the mean values for the "high prices" and the "low prices," respectively. As n increases, consumers categorize the firm's prices into finer and finer partitions. Therefore, their price recall will more closely reflect the actual price they previously observed.

The sequence of moves of the players are as follows: In the first stage, given the number of categories n and the cutoffs $\{k_i\}_{i=1}^n$, the two firms simultaneously decide on their pricing strategies. Note that the cutoffs and the firms' pricing strategies determine the mean prices that will be recalled from each category. In the second stage, all consumers make their purchase decisions based on price realizations and the decision process described earlier. We solve for the symmetric Nash equilibrium of the game. In addition, we also examine the optimal cutoffs for the limited memory consumers ($\{k_i^*\}_{i=1}^n$) satisfying the requirement that the equilibrium surplus of the limited memory consumers is maximized.

Before we begin the analysis it is useful to briefly state the results of the case in which all the consumers who compare prices have perfect memory ($2\beta = 0$). This corresponds to the standard model of competition as in Varian (1980) and Narasimhan (1988). Denote by p_j and π_j firm j's equilibrium price and profit, and let $W_j(p) = Pr(p_j \geq p)$. There exists no pure strategy equilibrium and the unique symmetric price equilibrium employs completely mixed strategies. Define b which satisfies $\gamma = b(\gamma + 2\alpha)$. Thus $b = \frac{\gamma}{1-\gamma}$ and the price distribution in a symmetric mixed strategy equilibrium is given by,

$$W_j(p) = W^*(p) = \left\{ \begin{array}{lll} 0 & \text{if} & p > 1 \\ \frac{\gamma}{1-2\gamma}(\frac{1}{p} - 1) & \text{if} & 1 \geq p \geq b \\ 1 & \text{if} & b > p \end{array} \right\}. \tag{1}$$

Note that the equilibrium profits are $\pi_j = \gamma$, and the expected equilibrium price is given by $E(p_j) = \frac{\gamma}{1-2\gamma} \ln(\frac{1-\gamma}{\gamma})$.

3. Analysis of Limited Memory and Price Competition

In this section, we present the equilibrium solution of the model and discuss the results. We start by analyzing the simple case of $n = 0$ in which the limited memory consumers cannot divide the price range into categories. We then consider the general case in which the limited memory consumers use n categories. Finally, we examine the limiting case of a market where the size of the perfect memory consumer group becomes vanishingly small. This enables us to focus on the effect of limited memory and its consequences for the market equilibrium.

3.1. The Case of No Memory, $(n = 0)$

If the limited memory consumers are unable to divide the price range into categories, they can only recall the mean price charged by the first firm in their decision process. Note that there is no pure strategy equilibrium for the firms' pricing decisions.[8]

Lemma 1: *The support of the symmetric equilibrium distribution contains no atoms and is comprised of two separate intervals with a hole in the middle. Specifically the price support is* $(b, m) \cup (v, 1)$ *where* m *denotes the mean of the distribution.*

Proof: Let $W_j(p) = Pr(p_j \geq p)$ be the probability that firm $j = 1, 2$ charges a price above p. In a symmetric equilibrium $W_j(p) = W(p)$. From standard arguments as in Varian (1980), the distribution function will have no mass points in the equilibrium support. The demand for a firm whose price approaches m from below in a symmetric equilibrium will be $\gamma + \beta + W(m)(2\alpha + \beta)$. Next for the price $m + \epsilon$, (for $\epsilon \to 0$), the firm's demand changes discontinuously to $\gamma + W(m)(2\alpha + \beta)$. Therefore, any such price will be dominated by m, implying that the equilibrium distribution will have a hole from m upto some $v > m$. All the remaining prices in the intervals (b, m) and $(v, 1)$ are part of the equilibrium support due to the same reason as in Varian (1980) or Narasimhan (1988). Q.E.D.

Note that there is no probability density between m and v. Therefore, we denote $w = W(v) = W(m)$. For the four extreme points of the distribution we get the following profit expressions. In any mixed strategy equilibrium all the prices in the support should yield the firm the same expected

[8] The reasons are: i) if the price of say Firm 1 is high enough, Firm 2 will have incentive to undercut the price to attract the 2α and/or 2β segments; ii) in the other cases, Firm 2 will have the incentive to increase its price to the reservation price, 1, and sell to just the γ consumers.

profit:

$$p = 1: \quad \Pi = \gamma + w\beta \tag{2}$$

$$p = v: \quad \Pi = (\gamma + w\beta + 2w\alpha)v \tag{3}$$

$$p = m: \quad \Pi = (\gamma + w\beta + \beta + 2w\alpha)m \tag{4}$$

$$p = b: \quad \Pi = (\gamma + w\beta + \beta + 2\alpha)b \tag{5}$$

When pricing at 1, a firm will get all of its uninformed consumers plus the limited memory consumers that started with it, recall m (rather than the actual price), and encounter a higher price higher than m when at the other firm. When charging v a firm will get in addition to the above consumers all the informed consumers with perfect memory that find a higher price in the other firm. The last two profit expressions can be derived in a similar fashion. Extending our analysis to any price in the equilibrium support and requiring that the profit is a constant Π we get the equilibrium pricing strategy as:

$$W_j(p) = W(p) = \begin{cases} 0 & \text{if} \quad p > 1 \\ \frac{\Pi}{2\alpha}\frac{1-p}{p} & \text{if} \quad 1 \geq p \geq v \\ \frac{\Pi}{2\alpha}\frac{1-v}{v} & \text{if} \quad v \geq p \geq m \\ \frac{\Pi}{2\alpha}\frac{1-p}{p} - \frac{\beta}{2\alpha} & \text{if} \quad m \geq p \geq b \\ 1 & \text{if} \quad b > p \end{cases}$$

Recall that m is just the mean of the distribution and thus

$$m = \int p \frac{d}{dp}(1 - W(p))dp = \frac{\Pi}{2\alpha}\ln(\frac{m}{b}) + \frac{\Pi}{2\alpha}\ln(\frac{1}{v})$$

where $v = \frac{\gamma + w\beta}{\gamma + w\beta + 2w\alpha}$, $m = \frac{\gamma + w\beta}{\gamma + w\beta + \beta + 2w\alpha}$ and $b = \frac{\gamma + w\beta}{\gamma + w\beta + \beta + 2\alpha}$.

Combining the above equations we get the following implicit equation for w, the probability of charging a price above v.

$$\ln\left(\frac{(\gamma + w\beta + 2w\alpha)(\gamma + w\beta + \beta + 2\alpha)}{(\gamma + w\beta)(\gamma + w\beta + \beta + 2w\alpha)}\right) = \frac{2\alpha}{\gamma + w\beta + \beta + 2w\alpha} \tag{6}$$

It can be shown that (6) has a unique solution for w in the interval $\left[0, \frac{1}{2}\right]$.

Note that the limited memory consumers that observe a firm's price first, offer that firm an incentive to price above the mean (as they only recall the mean when they compare with the other firm's price). While the limited memory consumers that start with the other firm offer the opposite incentive of pricing at or below the mean. The firm reconciles these differing incentives differences by mixing between offering only very deep or relatively shallow sales.

In order to highlight the effect of limited memory consumers we investigate the limiting case where the size of the group of consumers with perfect memory becomes very small. As $\alpha \to 0$ we get $v = 1$, $m = b = \frac{\gamma}{\gamma+\beta} = 2\gamma$. So the two price intervals reduce to two mass points at $p = 1$ and $p = 2\gamma$. Also, it can be shown from (6) or from the distribution function $W(p)$ derived above that as $\alpha \to 0$ then $w \to 0$. Hence the price distribution collapses to a single point at $m = 2\gamma$ with an expected profit of γ.

3.2. The General Case

We now present the analysis of the model for the general case of $n+1$ categories. As before we solve for the symmetric equilibrium in mixed strategies. Before we derive the equilibrium price support, note that the potential price range of each firm is $(b, 1)$, where $b = \frac{\gamma}{1-\gamma}$ as given in the case of perfect memory. A firm will never set a price p_j below b because the maximum profit it can obtain is $p_j(\gamma + 2\beta + 2\alpha)$, which is lower than γ, the guaranteed profit it can obtain by setting $p_j = 1$ and selling only to its uninformed consumer group. Hence, the $n+1$ categorizations are all within $(b, 1)$. Given our notation, we denote $k_0 = b = \frac{\gamma}{1-\gamma}$. The following lemma identifies the equilibrium price support:

LEMMA 2: *The support of the price distribution in a symmetric mixed strategy equilibrium for each firm's pricing decision is:* $\bigcup_{i=1}^{n+1} [(b_i, m_i) \cup (v_i, k_i)]$ *where* $k_i > v_i > m_i > b_i > k_{i-1}$ *(i.e., there is also a hole in the distribution between* b_i *and* k_{i-1}*).*

PROOF: Let $W_j(p) = Pr(p_j \geq p)$ be the probability that firm $j = 1, 2$ charges a price above p. In a symmetric equilibrium $W_j(p) = W(p)$. As in the previous lemma, the equilibrium price support will have no mass points. The demand for a firm whose price approaches m_i from below in a symmetric equilibrium will be $\gamma + \beta W(k_{i-1}) + (2\alpha + \beta)W(m_i)$. Next for the price $m_i + \epsilon$, (for $\epsilon \to 0$), the firm's demand changes discontinuously to $\gamma + \beta W(k_i) + (2\alpha + \beta)W(m_i)$. Therefore, any such price will be dominated by m_i, implying that the equilibrium distribution will have a hole from m_i upto some $v_i > m_i$. Define the minimum price charged in the category i to be b_i. Therefore, by the definition of the mixed strategy equilibrium we should have $\pi(b_i) = b_i(\gamma + \beta W(v_i) + \beta W(b_i) + 2\alpha W(b_i)) = \pi(k_{i-1}) = k_{i-1}(\gamma + \beta W(v_{i-i}) + \beta W(b_i) + 2\alpha W(b_i))$. Now because $W(v_{i-i}) > W(v_i)$, it follows that $b_i > k_{i-1}$. Therefore, there is a hole in the distribution between b_i and k_{i-1}. The remaining prices in (b_i, m_i) and (v_i, k_i) are part of the equilibrium price support due to standard arguments as in Varian (1980) and Narasimhan (1988). Q.E.D.

Denote $w_i = W(v_i)$ and $s_i = W(b_i)$ for $i = 1, \cdots, n+1$, (and by definition $s_{n+2} = 0$). Note that s_i is just the probability of pricing in any of the categories between $i \cdots, n+1$. Note that in equilibrium firms will choose their strategies to make their rivals indifferent between their strategies. For the extreme points of the distribution we get the following profit expressions for $i = 1, \cdots, n+1$:

$$\begin{aligned} p_j = k_i &: \Pi = (\gamma + w_i\beta + s_{i+1}\beta + 2s_{i+1}\alpha)\, k_i \\ p_j = v_i &: \Pi = (\gamma + w_i\beta + s_{i+1}\beta + 2w_i\alpha)v_i \\ p_j = m_i &: \Pi = (\gamma + w_i\beta + s_i\beta + 2w_i\alpha)m_i \\ p_j = b_i &: \Pi = (\gamma + w_i\beta + s_i\beta + 2s_i\alpha)b_i \end{aligned} \tag{7}$$

When pricing at k_i a firm will get four groups of consumers: i) all of its uninformed consumers, ii) the informed consumers with perfect memory who find a higher price at the other firm, iii) the limited memory consumers that started with it, recall m_i (rather than the actual price), and encounter a higher price than m_i at the other firm, iv) and finally the limited memory consumers that begin with the other firm, observe a price above b_{i+1} and remember a price m_{i+1}. When charging v_i a firm will get in addition to the above consumers all the informed perfect memory consumers who find a higher price at the other firm. A price of m_i will get a firm the obvious uninformed and informed perfect memory consumers as well as *all* the limited memory ones that started with the other firm and saw a price higher then b_i (as they remember m_i, but will not purchase from the first firm even in a case of a tie) and the limited memory consumers that started with it and compare to a price above m_i in the other firm. Finally by pricing at the lower end of the support a firm will get additional informed consumers with perfect memory as well. To solve for the equilibrium recall that m_i is the mean of the price distribution within category i and given by $m_i = \int_{b_i}^{k_i} p \frac{d}{dp}(1 - W(p))dp$.

A firm pricing at $k_{n+1} = 1$ guarantees itself a profits of $\pi = \gamma + w_{n+1}\beta$ which is the profit in the symmetric mixed strategy equilibrium. Therefore, firms make higher profits in a market with a strictly positive mass of limited memory consumers ($\beta > 0$). In the extreme case where all the informed consumers have perfect recall ($\beta \to 0$ and $\alpha > 0$), the equilibrium profits of the firms will only be γ. Thus the firms can take advantage of the co-existence of the limited memory consumers and the perfect memory consumers to soften competition and to achieve higher equilibrium profits. The presence of consumer heterogeneity in memory capacity leads to increased profits by softening the price competition between the firms.

3.3. The Equilibrium as $\alpha \to 0$

The effect of limited memory on competition can be clearly seen from the analysis of the limit market in which all the informed consumers have limited memory of degree n. If we take the limit of α approaching zero, then similar to section 3.1 we get $v_i = k_i$, $m_i = b_i$, and $w_i = s_{i+1}, i = 1, \cdots, n$

(as well as $w_{n+1} = 0$). In other words, as α approaches zero the two price support intervals in each category i shrink to two points $p = k_i$ and $p = m_i$ where the k_i's are not charged with positive probability. Consequently, the set of equations in (7) above reduces to:

$$p_j = m_i : \pi = (\gamma + s_{i+1}\beta + s_i\beta)m_i \quad \forall i = 1, \cdots, n+1 \tag{8}$$

A few comments about the nature of the equilibrium price support are in order. In the models of competitive price promotions with only perfect recall consumers the equilibrium price distribution is continuous (for example, Narasimhan 1988, Raju et. al. 1990, or Lal and Villas-Boas 1998). However, bounded rationality in the form of the ability to recall only the categories in which prices fall rather than the actual prices leads to firms choosing from only a finite set of possible prices.[9] Another point is that the number of prices charged goes up with the improvement in the degrees of memory. Behavioral studies have pointed out that high involvement environments lead to greater attentional capacity being devoted to encode a relevant piece of information in memory (e.g., Celsi and Olson 1988). Thus one can expect to observe more prices being charged in high involvement product-markets with greater degrees of consumer memory. The following proposition states the equilibrium of the limit market case:

Proposition 1: *In the limit market (as $\alpha \to 0$) with uninformed consumers and limited memory consumers who have n degrees of memory, a symmetric mixed strategy equilibrium will involve each firm charging prices at $p_i = m_i = \frac{2}{\frac{1}{k_{i-1}} + \frac{1}{k_i}}$ with probabilities $\Pr(m_i) = \frac{\gamma}{2\beta}(\frac{1}{k_{i-1}} - \frac{1}{k_i})$, for $i = 1, \cdots, n+1$ where the $\{k_i\}_{i=1}^n$ are the cutoffs the limited memory consumers are endowed with such that $1 = k_{n+1} > k_n > \cdots > k_1 > k_0 = \frac{\gamma}{1-\gamma}$.*

Proof: Note that for every positive α the support of the equilibrium price distribution is comprised of two intervals in each category and each price in the support leads to the same profit. Thus at the limit as $\alpha \to 0$ the profit from charging m_i must equal the profit from charging k_i $\forall i = 1, \cdots, n+1$. Charging a price of $p = k_{n+1} = 1$ gives a profit of γ which is the equilibrium profit. When charging $p = k_i$ the expected profit for a firm is

$$\pi_i = (\gamma + 2s_{i+1}\beta)k_i = \gamma \quad i = 1, 2, \cdots, n \tag{9}$$

[9]In empirical studies it has been observed that the distributions of prices are typically such that most of the probability mass is concentrated around a small number of price points (see for example, Villas-Boas 1995 and Rao et. al. 1995).

From this we can show that

$$s_i = \frac{\gamma(1-k_{i-1})}{2\beta k_{i-1}} \quad i = 2, 3, \cdots n+1, \tag{10}$$

and $s_1 = \frac{\gamma(1-k_0)}{2\beta k_0} = 1$ by definition. By noting that $Pr(m_i) = s_i - s_{i+1}$ we get:

$$\Pr(m_i) = \frac{\gamma(1-k_{i-1})}{2\beta k_{i-1}} - \frac{\gamma(1-k_i)}{2\beta k_i} = \frac{\gamma}{2\beta}(\frac{1}{k_{i-1}} - \frac{1}{k_i}) \tag{11}$$

In order to calculate the values of the prices charged in each category we use (8)

$$m_i = \frac{\gamma}{\gamma + s_i\beta + s_{i+1}\beta} = \frac{\gamma}{\gamma + \frac{\gamma(1-k_{i-1})}{2k_{i-1}} + \frac{\gamma(1-k_i)}{2k_i}} = \frac{2}{\frac{1}{k_{i-1}} + \frac{1}{k_i}} \tag{12}$$

Q.E.D.

Note that the prices charged in each category are the harmonic means of the cutoff prices. We had earlier noted that the profits for the case where all the informed consumers had perfect memory was γ. As seen from the proof of Proposition 1, the equilibrium profits in a market where all the informed consumers compare prices with limited memory is also γ. Furthermore, in this market with only limited memory consumers, the number of prices charged by firms is equal to the number of categories. Therefore, in the limit market equilibrium, the available memory capacity is aligned with the price information that is required to be recalled and it is as if a market with perfect recall is mimicked. This results in the firms competing away all but the guaranteed profits that can be made from their uninformed consumers. A final point worth mentioning is that the above equilibrium can be viewed as the result of search with an optimal stopping rule by the limited memory consumers. The firms' equilibrium pricing strategy coupled with costless consumer search guarantee that consumers can never lose by searching a second firm. The consumers will return to the original firm precisely when it is profitable for them to do so.

Proposition 1 identifies the equilibrium firm strategies as function of the given cutoffs of the limited memory consumers. We now derive the optimal cutoffs satisfing the requirement that the equilibrium surplus of the limited memory consumers is maximized. As shown in Proposition 2 below, these optimal cutoffs k_i^* also result in a symmetric equilibrium for consumers if they choose the cutoffs strategically. Note that each firm's equilibrium profits are always γ. Therefore, such a symmetric equilibrium is also uniquely Pareto optimal. The idea that consumers might do the best for themselves given the constraints that they face is in the spirit of bounded rationality as defined by Simon (1987) where the consumer makes a fully rational choice that takes into account her cognitive limitation. This also reflects an important aspect of human cognition: While we are limited in memory capacity, we can be sophisticated in identifying patterns and forming optimal decision rules.[10]

PROPOSITION 2: *As* $\alpha \to 0$, *the optimal cutoff prices that maximize the equilibrium surplus of the limited memory consumers are* $k_i^* = (\frac{\gamma}{1-\gamma})^{\frac{n+1-i}{n+1}}$. *For these cutoffs, each firm's equilibrium prices are* $m_i^* = \frac{2}{1+(\frac{1-\gamma}{\gamma})^{\frac{1}{n+1}}}(\frac{\gamma}{1-\gamma})^{\frac{n+1-i}{n+1}}$ *with probability* $\Pr(m_i^*) = \frac{\gamma}{1-2\gamma}[(\frac{1-\gamma}{\gamma})^{\frac{1}{n+1}} - 1](\frac{1-\gamma}{\gamma})^{\frac{n+1-i}{n+1}}$ *for* $i = 1, \cdots, n+1$. *No individual consumer will have the incentive to deviate from* k_i^* *given that the other consumers are also using these cutoffs.*

PROOF: Clearly the total surplus is $S = 1$ (a unit mass of consumers with a common reservation price of 1 and zero marginal cost of production). The total producer surplus of the two firms is $2\pi = 2\gamma$. The consumer surplus of the uninformed consumers is given by $S_\gamma = 2\gamma[1 - \sum_{i=1}^{n+1} \Pr(m_i)m_i]$. So the limited memory consumers' surplus will be, $S_\beta = S - S_\gamma - 2\pi = 1 - 4\gamma + 2\gamma \sum_{i=1}^{n+1} \Pr(m_i)m_i$. From (11) and (12) we can see that the expected price paid by the uninformed consumers is just $E_n(p) = \sum_{i=1}^{n+1} \Pr(m_i)m_i = \frac{\gamma}{\beta}\sum_{i=1}^{n+1}\frac{k_i - k_{i-1}}{k_i + k_{i-1}}$. Hence the necessary condition for maximizing S_β is that for $i = 1, \cdots, n+1$ we have $\frac{\partial S_\beta}{\partial k_i} = \frac{2\gamma}{\beta}\frac{(k_{i-1}-k_{i+1})(-k_{i-1}k_{i+1}+k_i^2)}{(k_i+k_{i-1})^2(k_{i+1}+k_i)^2} = 0$.[11] This implies that the condition for a maximum is $-k_{i-1}^* k_{i+1}^* + k_i^{*2} = 0$. So the cutoff prices $\{k_i^*\}_{i=1}^n$ form a geometric sequence with the boundary conditions $k_{n+1}^* = 1$, $k_0^* = \frac{\gamma}{1-\gamma}$. Hence for $i = 1, \cdots, n$ we have the optimal cutoffs to be:

$$k_i^* = (\frac{\gamma}{1-\gamma})^{\frac{n+1-i}{n+1}} \tag{13}$$

Substituting equation 13 into 12 and 11 yields the desired expression in the proposition for m_i^* and $Pr(m_i^*)$. Next, we can show that no consumer has the incentive to deviate from the optimal $\{k_i^*\}_{i=1}^n$ given that other consumers having $\{k_i^*\}_{i=1}^n$ as given above. The reasons are as the follows: If a consumer deviates from the $\{k_i^*\}_{i=1}^n$ given above, firms' pricing strategy will not change because of the assumption that there is a large number of consumers in the market. Then if such a deviation results in C_i containing either one or two prices from $\{m_i^*\}_{i=1}^{n+1}$ being charged at equilibrium, the consumer will have the same surplus. However, if such deviation leads C_i to contain three prices from$\{m_i^*\}_{i=1}^{n+1}$ (denoting them as $m_h > m_m > m_l$), the consumer can never be better off. Denote the new mean price of C_i to be $\overline{m}$. If $\overline{m} = m_m$, the consumer is indifferent. If $\overline{m} > m_m$, the consumer who observes m_l at the first firm will recall it as $\overline{m}$ and will buy from the second firm if the price there is m_m. If $\overline{m} < m_m$, the consumer who observes m_h at the first firm will recall it as $\overline{m}$ and will not buy from the second firm if the price there is m_m. In both cases, the consumer is worse off. Similarly, the consumer can never be better off if a deviation leads to C_i to contain more than three prices

[10]For example, an analyst is unlikely to be able to remember numbers in a data set but can be good at detecting patterns in the data and forming rules or heuristics and in using those rules.

[11]It is tedious but straight forward to calculate the Hessian, $\frac{\partial S_\beta^2}{\partial k_j \partial k_i}$, and show that the necessary conditions are indeed sufficient.

from$\{m_i^*\}_{i=1}^{n+1}$. Hence, an individual consumer has no incentive to deviate given other consumers' strategy. Q.E.D.

We can now summarize the main findings from the analysis of the limit market and compare these results with the perfect memory model. Even though consumers have limited memory, the pricing strategies of the firms adjust so that the number of prices charged is aligned with the degrees of consumer memory and so it is as if consumers can perfectly recall the actual prices that are charged. Thus the market equilibrium adjusts to the memory capacity of consumers and each firm charges only a single price with positive probability in each category. Additionally, an individual consumer has no incentive to deviate from the $\{k_i^*\}_{i=1}^n$ used by all other consumers because of the assumption that the market includes a large number of consumers, so that no individual consumer's deviation can change the firms' pricing strategies.

Next we can see from Proposition 2 that for $i = 1, \cdots, n+1$ the difference between consecutive cutoff points is:

$$k_i^* - k_{i-1}^* = (\frac{\gamma}{1-\gamma})^{\frac{n+1-i}{n+1}} - (\frac{\gamma}{1-\gamma})^{\frac{n+2-i}{n+1}} = (\frac{\gamma}{1-\gamma})^{\frac{n+1-i}{n+1}} \left(1 - (\frac{\gamma}{1-\gamma})^{\frac{1}{n+1}}\right) \tag{14}$$

The difference decreases as i decreases. This implies that the categorization becomes finer towards the lower end of price range. Furthermore, the probability of charging a particular price is proportional to $(\frac{1-\gamma}{\gamma})^{\frac{n+1-i}{n+1}}$ and thus is decreasing in i. These results are intuitively appealing as they suggest that the consumers pay more attention in encoding lower prices than higher prices and this induces firms to respond by charging the lower prices with higher probabilities. Also, we have that $\frac{\partial k_i^*}{\partial \gamma} > 0$. The level values of the cut-offs increase in less competitive markets with more uninformed consumers. As expected an increase in the size of the uninformed group of consumers increases each price that the firms will charge and also shrinks the price range $1 - b$.

3.4. Comparing Limited Memory to Perfect Memory

We now turn to the comparison of this model given optimal cutoffs to the standard model where all the informed consumers who compare prices are assumed to have perfect recall. As the degree of memory increases the price distribution resembles the standard model and in the limit as the degree of memory increases beyond bound it is identical to the distribution in equation (1). Formally,

PROPOSITION 3: *Given an arbitrarily small* $\delta > 0$, $\exists\ N$ *such that for every* $n > N$ *and every price* $p \in (\frac{\gamma}{1-\gamma}, 1)$ *we have i)* $\exists\ m_j(n)$ *such that* $|p - m_j(n)| < \delta$ *and ii)* $|W_n(p) - W(p)| < \delta$, *where* $W_n(p)$

is the equilibrium probability of each firm pricing above p *and* $m_j(n)$ *is a price charged with positive probability when the limited memory consumers employ* n *cutoff prices.*

Proof: When consumers' degree of memory is n, the maximum size of a category is $|1-k_n(n)| = 1-(\frac{\gamma}{1-\gamma})^{\frac{1}{n+1}}$ (recall that the categories are finer towards the lower end of the price range). Thus for $n > N_1 = \frac{\ln(\frac{\gamma}{1-\gamma})}{\ln(1-\delta)} - 1$, we have that the size of the largest category is smaller than δ. Let $p \in C_j(n)$ then the mean of the prices in this partition, $m_j(n) \in C_j(n)$ is charged with positive probability and is δ close to p. Recall from equation (10) that the probability of a firm pricing in category $C_j(n)$ or higher is just $s_j(n) = \frac{\gamma}{1-2\gamma}\left(\frac{1}{k_{j-1}(n)} - 1\right)$ where $k_{j-1}(n)$ is the lower cutoff bound of $C_j(n)$. Hence, if $p < m_j(n)$ we have $W_n(p) = s_j(n)$ otherwise $W_n(p) = s_{j+1}(n)$. Recall from equation (1) that $W(p) = \frac{\gamma}{1-2\gamma}(\frac{1}{p} - 1)$, so we have (for $p < m_j(n)$) $|W_n(p) - W(p)| = |s_j(n) - W(p)| = \frac{\gamma}{1-2\gamma}\left|\frac{1}{k_{j-1}(n)} - \frac{1}{p}\right| = \frac{\gamma}{1-2\gamma}\frac{|p-k_{j-1}(n)|}{k_{j-1}(n)p} < \frac{\gamma}{1-2\gamma}\frac{\delta}{p(p-\delta)} < \frac{\gamma}{1-2\gamma}\frac{\delta}{\frac{\gamma}{1-\gamma}(\frac{\gamma}{1-\gamma}-\delta)}$. If $p > m_j(n)$ then $|W_n(p) - W(p)| = |s_{j+1}(n) - W(p)| = \frac{\gamma}{1-2\gamma}\left|\frac{1}{k_j(n)} - \frac{1}{p}\right| < \frac{\gamma}{1-2\gamma}\frac{\delta}{p\times p} < \frac{\gamma}{1-2\gamma}\frac{\delta}{p(p-\delta)}$. Define η such that $\frac{\gamma}{1-2\gamma}\frac{\eta}{\frac{\gamma}{1-\gamma}(\frac{\gamma}{1-\gamma}-\eta)} < \delta$, so for any $n > N_2 = \frac{\ln(\frac{\gamma}{1-\gamma})}{\ln(1-\eta)} - 1$ we get that $|W_n(p) - W(p)| < \delta$. Finally, $N = \max(N_1, N_2)$. Q.E.D.

Thus we recover the standard model of perfect recall competition as the limiting case of our model with infinite degrees of memory. Next consider a convergence measure that is based on the expected price consumers pay. Define the degree of convergence by $\Delta E_n(p) = \frac{E(p)-E_n(p)}{E(p)}$, where $E(p)$ is the average equilibrium price in the case of perfect memory.

$$\Delta E_n(p) = 1 - \frac{2(n+1)}{\ln(\frac{1-\gamma}{\gamma})}\frac{[(\frac{1-\gamma}{\gamma})^{\frac{1}{n+1}} - 1]}{[(\frac{1-\gamma}{\gamma})^{\frac{1}{n+1}} + 1]} \tag{15}$$

Smaller values of this measure imply greater convergence of the expected price to the case of perfect memory. It is straight forward to verify that $\frac{\partial \Delta E_n(p)}{\partial n} < 0$ and $\frac{\partial^2 \Delta E_n(p)}{\partial n^2} > 0$. Therefore, the convergence is increasing in n but the marginal gain in convergence is decreasing in n. Thus additional categories have a lower effect in changing the expected price as compared to the first few categories. Also, we have that $\frac{\partial \Delta E_n(p)}{\partial \gamma} < 0$ and $\frac{\partial^2 \Delta E_n(p)}{\partial \gamma^2} > 0$, which implies that the convergence is faster (with the marginal gain in convergence decreasing) in a less competitive market that has a greater proportion of uninformed consumers. An increase in the proportion of the uninformed consumers means fewer comparison shoppers who have limited memory. This implies that the expected prices in the case of limited memory approach that for the case of perfect memory. In the extreme case, at $2\gamma = 1$, the difference disappears. It is easy to also verify that $\lim_{n\to\infty}(\Delta E_n(p)) = 0$. Overall, this categorization model of limited memory suggests that given strategic market interactions, small amounts of improvements in rationality can lead to equilibrium choices that are close to the perfect rationality outcome. As discussed previously, the strategic response of firms to limited memory is to charge exactly the same number of prices as the number categories that consumers encode in memory. In equilibrium, it is as if consumers have perfect recall for the equilibrium prices.

We can also examine the expected consumer welfare loss due to limited memory. Since $S_\beta(n) = S - S_\gamma - 2\pi = 1 - 4\gamma + 2\gamma E_n(p)$, we can define the relative loss in surplus that the comparison shopping consumers incur using $n+1$ categorizations versus perfect memory as $\lambda_n = \frac{S_\beta - S_\beta(n)}{S_\beta} = \frac{2\gamma(E(p) - E_n(p))}{1-4\gamma+2\gamma E(p)}$. Again it is straight forward to show that $\frac{\partial \lambda_n}{\partial n} < 0$, $\frac{\partial^2 \lambda_n}{\partial n^2} > 0$, $\frac{\partial \lambda_n}{\partial \gamma} > 0$, and $\frac{\partial^2 \lambda_n}{\partial \gamma^2} > 0$. Therefore, the relative surplus loss for the limited memory consumers is decreasing in n, and the rate at which the surplus decreases is decreasing in n. Finally, the relative surplus loss for the limited memory consumers increases with the proportion of uninformed consumers in the market. Because $\frac{\partial^2 \lambda_n}{\partial n^2} > 0$, the relative marginal gain in surplus of the limited memory consumers from an increase in memory is decreasing with n. Overall this suggests that the expected welfare loss to consumers from limited memory of price recall is attenuated when we account for market competition between firms for these consumers.

In addition, note that $\frac{\partial S_\beta(n)}{\partial \gamma} < 0$, $\frac{\partial^2 S_\beta(n)}{\partial \gamma^2} > 0$. Thus for any given degree of memory, the surplus of the limited memory consumers increases as the market becomes more competitive. Hence, when the market is more competitive, consumers have greater value in allocating more memory resources for encoding and storing price information. Interestingly, this is also precisely the situation for which firms will be using more prices in the equilibrium support. This points to an empirically testable proposition: In more competitive product markets, consumers are likely to use more degrees of memory which should result in more observed prices being used by firms.

3.5. The Decision Whether to Compare Prices

The decision process of the limited memory consumers which we have considered so far implies as in Dow (1991) that the consumers necessarily have contact with both firms, but that the price information from one of the firms is imperfect. In this section, we provide an extension which allows consumers to decide whether or not to compare prices at all after observing the price at the first firm. We show that all the results of the paper are unaffected in this extension. This extension can be viewed as being consistent with the interpretation of the decision process as search with optimal stopping. Consumers after observing the price at the first firm decide whether to stop or to search (with zero incremental search cost) and obtain the price at the second firm.

If the decision process involves the choice of whether or not to go to the second firm, then the consumer upon observing the price at the first firm will have to decide whether to stop and buy at that firm, or to compare prices by obtaining the price at the second firm. The consumer might optimally decide not to compare prices if the benefit of obtaining the price at the second firm is sufficiently small. This can occur in this model if the limited memory consumers encounter a sufficiently low price at the first firm so that (even with zero search costs) they would be worse off going to the second firm.

In the limit market case of $\alpha \to 0$ it immediately follows that the consumer is never worse off by deciding to obtain the price from the second firm after having observed a price at the first firm. Note from Proposition 1 that firms charge only the prices m_i with positive probability in each category. Thus if the limited memory consumer encounters a price in any category $i > 1$ she will be strictly better off going to the second firm to obtain its price given zero incremental cost of search. And for the lowermost category $i = 1$ the consumer will be no worse off. Therefore, the consumer will always have the incentive to search and obtain the price at the second firm in the decision process after having observed the price at the first firm and consequently all the results discussed above for the limit market case are not affected even if the consumer explicitly chooses whether or not to compare prices. Thus for the limit market case the consumers can be viewed as if they are engaged in search with optimal stopping.

Consider now the extension with $\alpha > 0$. For the case of no categories ($n = 0$), which is analyzed in the Appendix, there is a threshold price u below which the limited memory consumers will not compare prices with the second firm after observing the price at the first firm. If they indeed do decide to obtain the price at the second firm, they will encode the first price as a higher price $\hat{m}$ which is the mean price conditional on $p > u$. If the consumer is at the second firm, then she will recall the first firm's price as the conditional mean $\hat{m}$. The equilibrium support will be $(b, u) \cup (d, \hat{m}) \cup \ (v, 1)$ (where $b < u < d < \hat{m} < v < 1$). From the profit expressions at the extreme points of the distribution and from the definition of the conditional mean we can derive the equilibrium of this model. Finally, for the general case of n categories and $\alpha > 0$, in the lowermost category $i = 1$ the equilibrium price support is similar to that described above with a threshold price u above which the consumer will obtain the price at the second firm. Then as in the case of $n = 0$ the consumer will compare the price at the second firm with $\hat{m}$ the recalled price at the first firm. Interestingly, for this general case, we can show that in equilibrium consumers will use a threshold for only the lowest category $i = 1$ and therefore the main results of section 3.2 and 3.3 will hold. For example, as before the equilibrium profits with consumer heterogeneity in memory capacity are higher than when all the informed consumers are homogenous in their memory capacity (i.e., only perfect memory or only limited memory consumers).

4. Categorization Schemes and Robustness of Results

In this section we examine the robustness of the main results under different market conditions and categorization processes. While the actual pricing strategies of firms can obviously be driven by the specific features of the consumer decision process as well as specific categorization process, we demonstrate below a remarkable degree of robustness in the general effects of price categorization in competitive market settings.

4.1. Alternative Categorization Process: Comparing Category Labels

We first consider an alternative categorization process by which the limited memory consumers encode the prices from both firms as categories and compare the labels of the categories that the prices from both the firms fall into. Consumers buy from the firm whose recalled price was in a lower category. In the case of a tie, (i.e., if the recalled prices of the two firms are in the same category), then consumers purchase randomly from either firm with equal probability. We solve for the symmetric Nash equilibrium of this game which consists of a pricing strategy chosen by each firm and the set of optimal cutoffs chosen by the consumers. In cases where there are multiple equilibria for firms' pricing strategies given a set of cutoffs $\{k_i\}_{i=1}^{n}$, we assume the selection criterion that firms will play the equilibrium strategies with Pareto dominant payoffs for them.

We first derive the equilibrium for a market consisting of only limited memory consumers and then similar to the main analysis of the limit market in section 3.3 we analyze a market with both limited memory and uninformed consumers. The detailed analysis of the equilibrium when consumers decide by comparing category labels is presented in Proposition 4 shown in the Appendix. In what follows we discuss the main results and their robustness.

As shown in Proposition 4 in the Appendix, when the market consists of only limited memory consumers, there is a unique symmetic equilibrium in pure strategies. Firms' equilibrium profits are positive and higher than that implied by Bertrand competition which would result if these consumers had perfect memory. Thus bounded rationality in the form of limited recall and the use of only the categories of the prices charged in the market moves firms away from the Bertrand competition outcome. Ever since Edgeworth (1925) there has been a literature on possible resolutions to the Bertrand paradox, the idea that undifferentiated firms facing a homogenous consumer market might still be able to price above marginal costs and earn positive equilibrium profits. These resolutions have typically focused on supply-side factors such as capacity constraints (Levitan and Shubik 1972, Kreps and Scheinkman 1983), or the nature of cost functions (Baye and Morgan 2002). Our result adds to this literature by showing the role for bounded rationality on the consumer side as a means to resolve the Bertrand paradox.

Next we analyze and the effect of consumer heterogeneity in firm preference as well as in memory capacity and present the results in Proposition 5 in the Appendix (which are analogous to Proposition 1 of the previous section). As can be seen from the analysis, the main results of the previous section are robust to this alternative categorization scheme. Specifically we find as in the previous section that, (1) categorization is finer towards the lower end of the price range, (2) a small initial improvements in recall move the market outcomes quickly to the perfect recall outcome, (3) consumer heterogeneity and the presence of uninformed/loyal consumers in the market increase the equilibrium profits of the firms, and (4) the effect of increasing n is to weakly reduce the equilibrium profits of the firms. Furthermore, as the number of categories increases beyond bound this equilibrium also converges to one in which the informed consumers have perfect memory as in the standard model of Varian (1980).

There is an interesting and intuitively reasonable difference between the effects of this alternative categorization process and the one studied in section 3. The categorization process in section 3 which consumers compare a category to actual prices can be seen as creating a more competitive and undifferentiated environment for firms than the alternative categorization process presented here. Our analysis shows that all else being equal, the categorization process of comparing labels of categories can lead to less competitive markets and higher equilibrium profits. The intuition behind this result is that consumers in the label-comparing case have less price information than in the case of our basic model where the actual price from one firm is used in purchase decisions.

4.2. Alternative Representations of the Category

We also investigated whether the findings of our model are robust to different representations of the category. In section 3 we assumed that consumers represent the recalled category by the mean of the category and compare the mean to the actual price at the current firm. But we have also analyzed the model in which consumers represent the category by the median price in the category and we find that all the qualitative results of Section 3 are preserved even if consumers use the median rather than the mean. Further, the key results of Section 3 continue to hold even if consumers use other exogenous rules as representation of the category such as the top or the bottom of the category. Indeed, we find that the model where consumers remember the top of the category is mathematically equivalent to the categorization model of label comparison. Finally, it is interesting to note that when consumers use the mean of the category, their maximum surplus is actually higher than when consumers use exogenous rules such as the bottom, middle or the top of the category. This is because the mean of the category is endogenous to the equilibrium actions of the firm. Thus if consumers in a market were to learn through experience over time to do the best for themselves and were motivated to make the best possible purchase decisions, then the analysis suggests that they would learn to recall the category mean price rather than using an exogenous rule.

4.3. Comments on Robustness

The extensions above uncover a remarkable degree of robustness in the general effects of price categorization in competitive market settings. The first robust effect is about the manner in which consumers should categorize price so that their surplus is maximized. Across the different decision processes categorization schemes, different consumer heterogeneity conditions, and irrespective of whether the resulting equilibrium is one in pure or mixed strategies, we find that the categorization is finer towards the lower end of the price range. This suggests the general and robust effect that consumers who are bounded in their recall have a strategic motivation to devote more memory resources to encode lower prices in order to induce the firms to charge more favorable equilibrium prices.

Next, we recover a consistent convergence property: the interaction of optimal categorization and market competition is such that small initial improvements in recall move the market outcomes quickly to the perfect recall outcome. Thus the initial few increases in the number of categories lead to equilibrium pricing choices and consumer surplus which are close to the standard perfect recall case where the number of categories are infinite. Furthermore, as the number of categories increases beyond bound this equilibrium also converges to one in which the informed consumers have perfect memory as in the standard model of Varian (1980). As expected, we also find that consumer heterogeneity and the presence of uninformed/loyal consumers in the market increase the equilibrium profits of the firms irrespective of the type of categorization and market conditions. Finally, we also find that the effect of increasing n is in general to weakly reduce the equilibrium profits of the firms.

5. Summary and Discussion

In this paper, we take a preliminary step towards understanding the effects of limited memory and categorization on price competition. We focus on a specific aspect of memory limitations, namely, the ability of consumers to recall price information only in categories. The paper introduces price comparisons with limited memory in a competitive market and analyzes the interaction between consumer price categorization and the equilibrium pricing strategies employed by firms.

We find that presence of heterogeneity in the memory capacity in the market allows firms to soften competition resulting in greater equilibrium profits. When the market consists of both consumers who have perfect recall as well as consumers who have limited memory, the equilibrium profits are greater than in a market with only limited memory or only perfect memory consumers. To focus on the effect of imperfect memory we analyze the limit market consisting of only uninformed consumers and informed consumers with limited memory. The equilibrium price strategy of each firm is comprised of a finite number of prices with firms charging only one price per memory category with positive probability. Thus, the strategic behavior of firms adjusts to limited memory so that the number of prices charged is aligned with the degree of consumer memory. Furthermore, the probability of charging a price is decreasing in the price and hence lower prices are charged with higher probability. When optimally choosing their cutoff points consumers use finer categorization towards the bottom of the price distribution. As a result, even with very few cutoff prices the expected price consumers pay and their surplus is close to the perfect recall case. A key suggestion of this categorization model of memory is that markets have a way of adjusting for the memory limitations of consumers. We also show that the main results of our model regarding the interaction between consumers' optimal price categorization and firms' equilibrium pricing strategies are remarkably consistent across different categorization processes and market conditions.

There are several interesting questions that are related to our investigation of limited memory. The problem of allocation of limited memory to different tasks, such as recalling several product attributes or the prices of different products that the consumer buys seems to an interesting problem to pursue. In this paper, we model memory limitations as consumers being able to recall price information only in categories. Alternatively, imperfect memory can be modeled as consumers recalling a price with a random noise or consumers recalling a price distribution instead of its exact realization. It might also be useful to explore other memory mechanisms and their effects on a firm's decision making. Memory can also be thought of as a device to carry information over time. The information-theoretic characterization of memory is relevant in markets for frequently purchased goods across different shopping occasions. Finally, on the experimental side it would be interesting to understand how the distributional characteristics of market variables such as price or product quality would affect their encoding in consumer memory. Overall, the analysis of limited recall in market settings can be a fruitful area for future investigation.

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APPENDIX

The Decision to Compare Prices in the Basic Model

The asymmetric categorization model in the paper assumes that the limited memory consumers have contact with both the firms by assumption. We now present the analysis of the case in which the decision process of the limited memory consumers allows them to decide whether or not to compare prices at all after the price at the first firm is observed. Given the price that a consumer encounters at the first firm, the consumer has to decide whether or not or to continue and obtain the price from the second firm. If the consumer encounters a low enough price at the first firm then she might decide not to compare prices at the second firm.

The $n = 0$ case

We start with the case of $n = 0$. Denote by u the threshold price below which the limited memory consumers will not obtain the price at the other firm. Define $\hat{m}$ to be the mean price conditional on $p > u$. Then the equilibrium support will be $(b, u) \cup (d, \hat{m}) \cup \ (v, 1)$ where $b < u < d < \hat{m} < v < 1$. Define $W(p)$ as $Pr(p \geq p)$ and $f(p)$ as the p.d.f of price. We have that:

$$
\begin{aligned}
\pi &= \gamma + \beta W(\hat{m}) \quad (p = 1) \\
\pi &= [\gamma + \alpha W(v) + \beta W(\hat{m})]v \quad (p = v) \\
\pi &= [\gamma + (\alpha + \beta) W(\hat{m}) + \beta W(u)]\hat{m} \quad (p = \hat{m}) \\
\pi &= [\gamma + \alpha W(d) + \beta W(\hat{m}) + \beta W(u)]w \quad (p = d) \\
\pi &= [\gamma + \alpha W(u) + \beta + \beta W(u)]u \quad (p = u) \\
\pi &= [\gamma + \alpha + \beta + \beta W(u)]b \quad (p = b) \\
\pi &= [\gamma + \alpha W(p) + \beta W(\hat{m})]p \quad (v < p < 1) \\
\pi &= [\gamma + \alpha W(p) + \beta W(\hat{m}) + \beta W(u)]p \quad (d < p < \hat{m}) \\
\pi &= [\gamma + \alpha W(p) + \beta + \beta W(u)]p \quad (b < p < u)
\end{aligned}
$$

Using the above expressions and the facts that $W(\hat{m}) = W(v)$, $W(d) = W(u)$, $W(1) = 0$ and $W(b) = 1$ and defining $W(\hat{m}) = h$ and $W(u) = g$, we can compute that in the mixed strategy equilibrium the following hold:

$$
\begin{aligned}
W(p) &= \frac{\gamma + \beta h}{ap} - \frac{\gamma + \beta h}{a} \quad (v \leq p \leq 1) \\
W(p) &= \frac{\gamma + \beta h}{ap} - \frac{\gamma + \beta h + \beta g}{a} \quad (d \leq p \leq \hat{m}) \\
W(p) &= \frac{\gamma + \beta h}{ap} - \frac{\gamma + \beta + \beta g}{a} \quad (b \leq p \leq u)
\end{aligned}
$$

Therefore from the profit expressions at the extreme points in the distribution we have that $v = \frac{\gamma + \beta h}{\gamma + \alpha h + \beta h}$,

$\hat{m} = \frac{\gamma+\beta h}{\gamma+\alpha h+\beta h+\beta g}$, $d = \frac{\gamma+\beta h}{\gamma+\alpha g+\beta h+\beta g}$, $u = \frac{\gamma+\beta h}{\gamma+\alpha g+\beta+\beta g}$, $b = \frac{\gamma+\beta h}{\gamma+\alpha+\beta+\beta g}$. From the definition of $\hat{m}$ and the expressions for $W(\cdot)$ derived above, we have:

$$\begin{aligned}\hat{m} &= \frac{\gamma+\beta h}{a}[\int_v^1 \frac{1}{p}dp + \int_d^{\hat{m}} \frac{1}{p}dp] \\ &= \frac{\gamma+\beta h}{a}\ln(\frac{1}{v}\frac{\hat{m}}{d})\end{aligned}$$

Now from the definition of u and the expressions for $W(\cdot)$ derived, we can derive the optimal stopping rule which determines the threshold price below which the consumer will not compare prices.

$$\begin{aligned}0 &= \int_d^{\hat{m}} (u-p)f(p)dp + \int_b^u (u-p)f(p)dp \\ &\Rightarrow \ u(1-h) = \frac{\gamma+\beta h}{a}\ln\frac{\hat{m}}{d}\frac{u}{b}\end{aligned}$$

Then using the expressions of $v, \hat{m}, d, u, b$ derived earlier, we have

$$\begin{aligned}\frac{\gamma+\beta h}{\gamma+\alpha h+\beta h+\beta g} &= \frac{\gamma+\beta h}{a}\ln(\frac{\gamma+\alpha h+\beta h}{\gamma+\beta h}\frac{\gamma+\alpha g+\beta h+\beta g}{\gamma+\alpha h+\beta h+\beta g}) \\ \frac{\gamma+\beta h}{\gamma+\alpha g+\beta+\beta g}(1-h) &= \frac{\gamma+\beta h}{a}\ln(\frac{\gamma+\alpha g+\beta h+\beta g}{\gamma+\alpha h+\beta h+\beta g}\frac{\gamma+\alpha+\beta+\beta g}{\gamma++\beta+\beta g})\end{aligned}$$

The equilibrium (h, g) can be solved from the above equations given that by definition $g \geq h$. As $\alpha \to 0$, we have

$$\begin{aligned}\pi &= \gamma+\beta h \ ; \ \pi = [\gamma+\beta h]v \\ \pi &= [\gamma+\beta h+\beta g]m \ ; \ \pi = [\gamma+\beta h+\beta g]d \\ \pi &= [\gamma+\beta+\beta g]b \ ; \ \pi = [\gamma+\beta+\beta g]u\end{aligned}$$

And hence, $b = u, \hat{m} = d, v = 1$. Also as $\alpha \to 0$, $W(p) = \frac{\gamma+\beta h}{ap} - \frac{\gamma+\beta h}{a}$ $(v \leq p \leq 1)$ leads to $h = W(v) = \frac{\gamma+\beta h}{ap} - \frac{\gamma+\beta h}{a} = 0$. Therefore, based on

$$\frac{\gamma+\beta h}{\gamma+\alpha g+\beta+\beta g}(1-h) = \frac{\gamma+\beta h}{a}\ln(\frac{\gamma+\alpha g+\beta h+\beta g}{\gamma+\alpha h+\beta h+\beta g}\frac{\gamma+\alpha+\beta+\beta g}{\gamma+\alpha g+\beta+\beta g})$$

we can show that $g = 0$ and that this would result in the same solution for the case of the $n = 0$ case in the model of section 3. Thus, $\hat{m} = d = v = 1$ and all the probability mass is on u. Hence, when $\alpha \to 0$, the profit equation at u is the same as the profit function at m in the paper. Consequently, all the results of the paper at $\alpha \to 0$ will be preserved.

The General Case of n categories

In the general n-category case, for any $\alpha > 0$, we can prove that the stopping rule only applies to the lowest category in the equilibrium, i.e. $i = 1$. The proof is as follows. Suppose there exists some category $i > 1$, in which there is a threshold price u_i below which limited memory consumers do not compare prices with the other firm. Because $i > 1$, if the limited memory consumers do not search upon encountering a price below u_i, then the firms have the incentive to undercut each other for only the perfect memory consumers in that category. Therefore, (v_{i-1}, u_i) will be on the equilibrium support. This implies that $k_{i-1} \in (v_{i-1}, u_i)$ should be on the equilibrium support. However, consumers have incentive to search and compare prices at k_{i-1} but not at $k_{i-1} + \varepsilon$. Thus, there must be a mass point at k_{i-1} to make the profit at k_{i-1} equal to the profit at $k_{i-1} + \varepsilon$ as $\varepsilon \to 0$. However, a mass point at k_{i-1} can not be part of an equilibrium because the other firm will have incentive to undercut it with a mass point at $k_{i-1} - \varepsilon$. Hence, there is a contradiction, and so there is not an equilibrium in which the limited memory consumers do not compare prices for $i > 1$. Therefore, in equilibrium the threshold only applies to category 1 regardless of the size of α.

For category $i = 1$, the equilibrium price support is $(b_1, u) \cup (d, \hat{m}) \cup (v_1, k_1)$ where $b_1 < u < d < \hat{m} < v_1 < k_1$. $\hat{m}$ is the mean price and u is the threshold price below which the limited memory consumers will not obtain a price at the second firm. Define $g = W(d) = W(u)$, the profit equations for category 1 corresponding to (1) in the paper become

$$
\begin{aligned}
p_j &= k_1: \ \pi = (\gamma + w_1\beta + s_2\beta + 2s_2\alpha)k_1 \qquad \text{(i)} \\
p_j &= v_1: \pi = (\gamma + w_1\beta + s_2\beta + 2w_1\alpha)v_1 \\
p_j &= \hat{m}: \pi = (\gamma + w_1\beta + g\beta + 2w_1\alpha)\hat{m} \\
p_j &= d: \pi = [\gamma + w_1\beta + g\beta + 2g\alpha]d \\
p_j &= u: \pi = [\gamma + g\beta + \beta + 2g\alpha]u \\
p_j &= b_1: \pi = [\gamma + g\beta + \beta + 2\alpha]b_1
\end{aligned}
$$

As in the $n = 0$ case, when $\alpha \to 0$, we have $v_1 \to k_1$, $\hat{m} \to v_1$, $d \to \hat{m}$, $b_1 \to u$, and only (b_1, u) is charged with positive probability. Therefore, we can see that u is also the unconditional mean of category 1, i.e. $u = m_1$. Thus, when $\alpha \to 0$, equations (i) here become

$$
\begin{aligned}
p_j &= k_1: \ \pi = (\gamma + 2s_2\beta)k_1 \\
p_j &= m_1: \pi = [\gamma + s_2\beta + \beta]m_1
\end{aligned}
$$

The above equations are the same as those for $p_j = k_1$ and $p_j = m_1$ at $\alpha \to 0$ given in the paper (without the optimal stopping rule). Hence, all results in the paper are preserved.

Alternative Categorization Process: Comparing Category Labels

We now present the analysis of model in which the limited memory consumers compare the labels of the categories in order to make their purchasing decision. We first start with the simple case of a market with only limited memory consumers. The following Proposition presents the equilibrium:

PROPOSITION 4: *Consider a market with limited memory consumers with n degrees of memory. When consumers optimally choose the categories, there is a unique pure strategy equilibrium. The optimal cutoffs are $k_i^* = \left(\frac{1}{2}\right)^{n+1-i} + \varepsilon$, for every $i = 1, ..., n$, where $\varepsilon \to 0$ and $\varepsilon << \left(\frac{1}{2}\right)^n \forall n$, and both firms charge $p_j^* = k_1^* = (\frac{1}{2})^n + \varepsilon$. Each firm makes positive equilibrium profits $\pi_j^* = (\frac{1}{2})^{n+1} + \frac{\varepsilon}{2}$, which are decreasing in n.*

PROOF: Each firm can potentially use exactly $n+1$ prices conditional on the choice of the partitions by the consumers. Firm 1 will have demand only if its price is the same or lower than firm 2's price. Clearly both firms charging at the top of the lowest partition ($p_1 = p_2 = k_1$) is an equilibrium. A firm that raises price will lose all consumers while lowering the price will only bring lower revenues from half of the market. The payoff matrix for firm 1 is given by (the payoff for firm 2 can be specified in an analogous way):

$$\Pi_1 (p_1 = k_r, p_2 = k_t) = \begin{cases} 0 & \text{if} \quad r > t \\ \frac{k_r}{2} & \text{if} \quad r = t \\ k_r & \text{if} \quad r < t \end{cases} \qquad for \quad r, t = 1, ..., n+1 \tag{ii}$$

Consumers will optimally choose their cutoffs so that the unique equilibrium will be at lowest partition and that the lowest cutoff will be at the lowest possible value. We start by considering the highest possible prices. For $p_1 = k_{n+1} = 1$, $p_2 = k_{n+1} = 1$ not to be an equilibrium the cutoffs must satisfy $\frac{k_{n+1}}{2} < k_n$ similarly for $p_1 = k_n$, $p_2 = k_n$ not to be an equilibrium we need $\frac{k_n}{2} < k_{n-1}$. Similarly, we need $\frac{k_{r+1}}{2} < k_r$ for $r = 1, ..., n$. Iterated substitution leads to the following condition on k_r ($r = 1, ..., n$)

$$k_r > \left(\frac{1}{2}\right)^{n+1-r}$$

Clearly the best choices for consumers are $k_r^* = \left(\frac{1}{2}\right)^{n+1-r} + \varepsilon$ for any infinitesimal $\varepsilon > 0$. Both firms price at the top of the lowest partition and split the market generating profits of $\pi_j^* = \frac{1}{2}\left(\frac{1}{2}\right)^n + \frac{\varepsilon}{2}$. Furthermore, given the choice of all consumers no single consumer can benefit from unilaterally changing her cutoff points. Q.E.D.

Next, in the following Proposition we present the analysis when the market consists of limited memory consumers as well as uninformed consumers who shop at a favorite firm. We allow for 2γ consumers to randomly purchase with equal probability from one or the other firm, or equivalently that each firm has a segment of γ consumers who buy only from that firm.

PROPOSITION 5: *Consider a market with uninformed consumers of size 2γ who randomly purchase from either firm with equal probability, and $(1-2\gamma)$ limited memory consumers with n degrees of memory. When consumers optimally choose the categories and $(\frac{1}{2(1-\gamma)})^n > 2\gamma$, there is a unique symmetric pure strategy equilibrium. The optimal cutoffs are $k_i^* = \left(\frac{1}{2(1-\gamma)}\right)^{n+1-i} + \varepsilon$, for every $i = 1, ..., n$, where $\varepsilon \to 0$ and $\varepsilon << \left(\frac{1}{2(1-\gamma)}\right)^n \forall n$, and both firms charge $p_j^* = k_1^* = (\frac{1}{2(1-\gamma)})^n + \varepsilon$.*

Proof: If both firms price at the lowest cutoff $p_1 = p_2 = k_1$ each makes a profit of $k_1/2$. For any pair of prices the payoff matrix for firm 1 is given by (and analogously for Firm 2):

$$\Pi_1 \left(p_1 = k_r, p_2 = k_t\right) = \begin{cases} \gamma k_r & \text{if} \quad r > t \\ \frac{k_r}{2} & \text{if} \quad r = t \\ (1-\gamma) k_r & \text{if} \quad r < t \end{cases} \qquad for \quad r, t = 1, ..., n+1 \tag{iii}$$

The firm with the lower price gets all the consumers except those loyal to its rival. When both firms have equal prices they split the market. For $r = 1, ..., n$ the pure strategies $(p_1 = k_r, p_2 = k_r)$ constitute a strict equilibrium if no firm wants to deviate. The most profitable deviation for, say, Firm 1 is to charge the highest possible price of $p_1 = 1$ making a profit of $\pi_1 = \gamma$ (and the same holds for Firm 2). Therefore, no firm will have the incentive to deviate and charge the reservation price if,

$$\Pi_j \left(p_1 = k_r, p_2 = k_r\right) = \frac{k_r}{2} > \gamma \tag{iv}$$

And no firm has an incentive to lower the price to the category immediately below if,

$$\Pi_j \left(p_1 = k_r, p_2 = k_r\right) = \frac{k_r}{2} > (1-\gamma)\, k_{r-1} \tag{v}$$

For the top most category the equilibrium condition is only,

$$\Pi_j \left(p_1 = k_{n+1}, p_2 = k_{n+1}\right) = \frac{1}{2} > (1-\gamma)\, k_n \tag{vi}$$

While for the bottom category it is,

$$\Pi_j \left(p_1 = k_1, p_2 = k_1\right) = \frac{k_1}{2} > \gamma$$

If $\frac{k_1}{2} \geq \gamma$ and for every $r = 2, ..., n$ conditions (v) are not satisfied and condition (vi) is not satisfied as well we get that the unique symmetric equilibrium in pure strategies is for both firms to price at k_1. If the conditions (v, vi) are not satisfied we have for $r = 2, .., n+1$

$$\frac{k_r}{2} \leq (1-\gamma)\, k_{r-1}$$

This condition implies $\frac{1}{2} = \frac{k_{n+1}}{2} \leq (1-\gamma)\, k_n$ or $k_n \geq \frac{1}{2(1-\gamma)}$. Applying the above condition iteratively we get $k_r \geq \left(\frac{1}{2(1-\gamma)}\right)^{n+r-1}$ and finally $k_1 \geq \left(\frac{1}{2(1-\gamma)}\right)^{n}$. Adding the condition $\frac{k_1}{2} \geq \gamma$ guarantees that pricing at k_1 is an equilibrium for both firms. So we get the condition,

$$k_1 \geq \left(\frac{1}{2\,(1-\gamma)}\right)^{n} \geq 2\gamma$$

Since $\gamma < \frac{1}{2}$, we have $2\gamma \geq \frac{\gamma}{1-\gamma}$ so we are guaranteed that $k_1 > k_0 = \frac{\gamma}{1-\gamma}$.

We still need to check that no pure strategy non-symmetric equilibrium exists. Consider the payoff matrix (iii) if $r > t+1$ then the payoff to firm 2 is $(1-\gamma)k_t$ but a deviation to pricing at k_{t+1} will yield a profit of $(1-\gamma)k_{t+1}$ which is higher so in any potential pure strategy equilibrium firms must price at most one category apart. Now assume that Firm 1 prices at k_t and Firm 2 prices at k_{t+1} then firm 2 makes a profit of γk_{t+1}. If it lowers its price to k_t it would make $\frac{k_t}{2}$. If $\frac{k_t}{2} > \gamma k_{t+1}$ then pricing at k_t and k_{t+1} cannot be an equilibrium. But we know even more, we know $\frac{k_t}{2} > \gamma$ $(k_t \geq k_1 > 2\gamma)$ and so we are done.

Consumers will try to choose their strategies to induce as low a k_1 as possible so we get $k_r^* = \left(\frac{1}{2(1-\gamma)}\right)^{n+r-1} + \varepsilon$ for a very small $\varepsilon > 0$. Firms price at $p_1^* = p_2^* = \left(\frac{1}{2(1-\gamma)}\right)^n + \varepsilon$ and make a profit of $\frac{1}{2}\left(\frac{1}{2(1-\gamma)}\right)^n + \frac{\varepsilon}{2}$. Q.E.D.

Mixed Strategy Equilibrium

Consider the case of a market with limited memory consumers and uninformed consumers and investigate the case where $\left(\frac{1}{2(1-\gamma)}\right)^n < 2\gamma$. For this case we characterize a mixed strategy equilibrium which is as follows:

Given $\frac{\gamma}{1-\gamma} \leq k_1 \leq ... \leq k_n \leq k_{n+1} = 1$, we can characterize the completely mixed symmetric equilibrium to be given by the unique solution to the set of equations that will be derived below. Assume that each firm prices at k_j with probability q_j $j = 1,..,n+1$. Then the profit for firm j when pricing at k_j is given by $j = 1,..,n$

$$\Pi_i(k_j) = k_j \left(\gamma + \beta q_j + 2\beta \sum_{m=j+1}^{n+1} q_m\right)$$

Pricing at the $k_{n+1} = 1$ yields:

$$\Pi_i(k_{n+1}) = (\gamma + \beta q_{n+1})$$

While pricing at the k_1 yields:

$$\Pi_i(k_1) = k_1[\gamma + \beta(2 - q_1)]$$

For a totally mixed strategy equilibrium we need $\Pi_i(k_j) = \Pi = \textit{constant}$ for $j = 1,..,n+1$, as well as $\sum_{m=1}^{n+1} q_m = 1$. So we have $n+2$ equations with $n+2$ unknowns that possess a unique solution. Subtracting the equation for k_{j+1} from the one for k_j yields the following set of equations for $j = 1,..,n$

$$q_j + q_{j+1} = \frac{\Pi}{\beta}\left(\frac{1}{k_j} - \frac{1}{k_{j+1}}\right) > 0$$

The specific solution to this set of equations depends upon the parity of n. So suppose for instance that n is an odd number, then a possible solution can consist of consumers choosing $q_{j+1} = 0$, $j = n, n-2, n-4...$

The proof is as follows: If n is odd and $q_{j+1} = 0$, $j = n, n-2, n-4..$,then we have $\Pi = \gamma + \beta q_{n+1} = \gamma$, and

$$\begin{aligned} q_{n+1} &= 0 \\ q_n &= \frac{\Pi}{\beta}\left(\frac{1}{k_n} - \frac{1}{k_{n+1}}\right) = \frac{\gamma}{\beta}\left(\frac{1}{k_n} - 1\right) \\ q_{n-1} &= 0 \\ q_{n-2} &= \frac{\gamma}{\beta}\left(\frac{1}{k_{n-2}} - \frac{1}{k_{n-1}}\right) \\ &... \end{aligned}$$

Then

$$\begin{aligned} q_{n-1} + q_n &= \frac{\gamma}{\beta}\left(\frac{1}{k_{n-1}} - \frac{1}{k_n}\right) = \frac{\gamma}{\beta}\left(\frac{1}{k_n} - 1\right) \\ &\rightarrow \frac{1}{2k_n} = (1 + \frac{1}{k_{n-1}}) \end{aligned}$$

Similarly, we have

$$\begin{aligned} q_{n-3} + q_{n-2} &= \frac{\gamma}{\beta}\left(\frac{1}{k_{n-3}} - \frac{1}{k_{n-2}}\right) = \frac{\gamma}{\beta}\left(\frac{1}{k_{n-2}} - \frac{1}{k_{n-1}}\right) \\ &\rightarrow \frac{1}{2k_{n-2}} = (\frac{1}{k_{n-1}} + \frac{1}{k_{n-3}}) \\ &... \end{aligned}$$

where we can define $k_0 = \frac{\gamma}{1-\gamma}$ (the lower bound).

If we solve the whole system of equation with consumers' surplus maximization, we will get

$$\begin{aligned} k_i^* &= (\frac{\gamma}{1-\gamma})^{\frac{n'+1-i'}{n'+1}} \text{for } i = n+1, n-1, n-3... \\ \frac{1}{2k_{i-2}^*} &= (\frac{1}{k_{i-1}^*} + \frac{1}{k_{i-3}^*}) \text{for } i = n+2, n, n-2... \end{aligned}$$

where $n' = \frac{n+1}{2}$ and $i' = \frac{i+1}{2}$. We can then notice that k_i^* $(i = n+1, n-1, n-3...)$ are like the k_i^* in the asymmetric categorization model with limited memory and informed consumers and k_i^* $(i = n, n-2, n-4...)$ are like the m_i^* (means) in the asymmetric categorization model. It can also be easily noted that this solution gives consumers greater surplus that the pure strategy equilibrium. As we have shown in the asymmetric case, the above equilibrium converges to the perfect recall solution as in Varian (1980) and Narasimhan (1988) as the number of categories increases beyond bound.

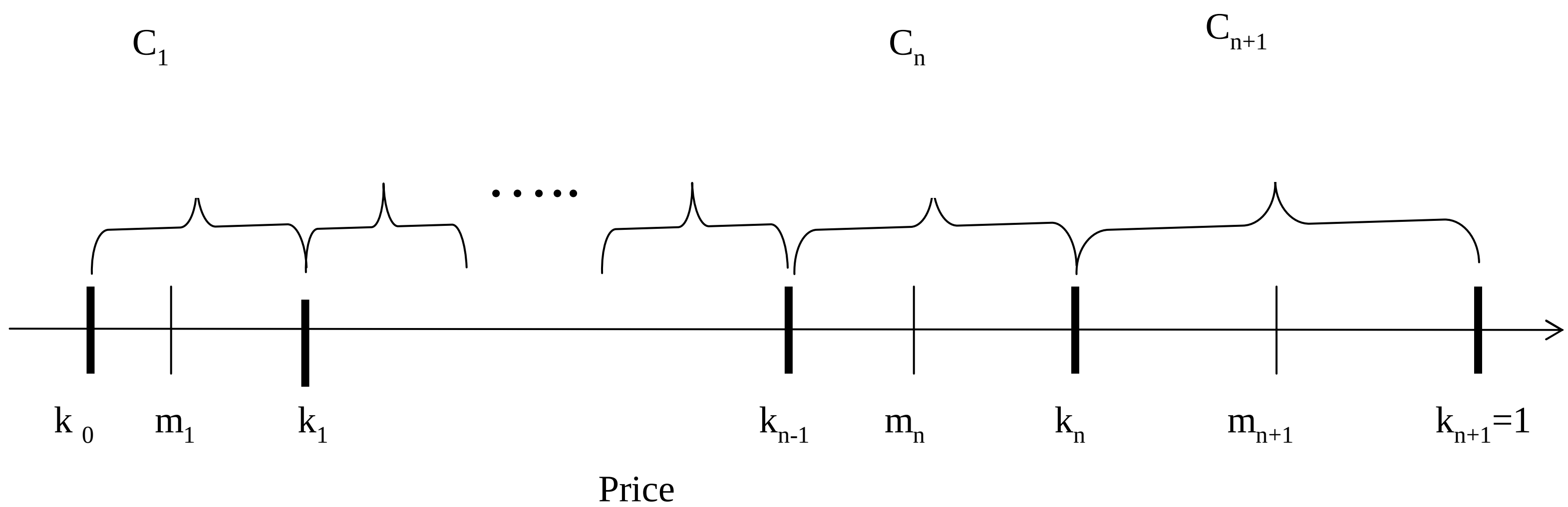

Figure 1: The Categorization Scheme