Monetary Policy Targeting and Economic Growth: An Equivalence Investigation

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Abstract

In a simple Ak endogenous growth model with flexible prices where a cash-in-advance constraint applies to both consumption and investment goods, we study the equivalent relation between money growth and interest rate rules. We restrict these monetary policy rules to yield the same balanced growth path equilibria, to exhibit the same equilibrium dynamics, and to have qualitatively equivalent comparative statics results. Under these equivalent criteria, we find that an active interest rate is equivalent to an active money growth rule, where the central bank raises its policy target by more than one percentage point in respond to a one-percentage point increase in inflation. When monetary policy rules are passive and the intertemporal elasticity of substitution in consumption is sufficiently large (greater than unity), real indeterminacy occurs and policy equivalence cannot be established. Finally, under logarithmic preferences, we find that constant money growth rules are identical to interest rate pegging rules.

Key Words: money growth rules, interest rate rules, equivalence.
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1 Introduction

The study of monetary policy has long been an important topic in macroeconomics. In his now-famous Carnegie-Rochester paper, Taylor (1993) summarizes the operation of monetary policy by a rule that adjusts the nominal interest rate in response to output and inflation, now widely known as the Taylor rule. Since then the recent literature on monetary policy has almost focused on interest rate rules exclusively [e.g., Benhabib et al]. However, most of the textbook cases in macroeconomics and monetary economics still describe monetary policy in terms of constant money growth rules advocated by Milton Friedman (1959) nearly half a century ago. In practice, monetary policies are usually stated in terms of the short-term interest rate targets (such as the Federal Fund rate in the U.S.) which are achieved by the central banks through their control over different monetary aggregates (e.g., the money supply or more accurately the supply of reserves). However, as pointed out by Eichenbaum (1992), the qualitative and quantitative effects of monetary policy are sensitive to the choice of measures of disturbances to monetary policy (i.e., innovations to short-term interest rates versus innovations to central bank reserves) used in the vector autoregressive analysis. On the other hand, based on the quantity equation, Taylor (1999) argued that a feedback interest rate rule "provides a good description of monetary policy in a fixed money growth regime." Empirically, Fève and Auray(2002) and Minford et. al. (2002) have both shown that the estimated interest rate relation with output and inflation of a Taylor rule is observationally equivalent to the equilibrium relation of the same variables in a monetary economy with exogenous money growth rule. Thus, it is worth investigating the actual relation between these two types of monetary policy rules.

Recently, several papers have attempted to study how money growth and interest rates are related and whether equivalence can be established among simple rules for both. Carlstrom and Fuerst (1995) compares the welfare effects of these two types of monetary policy rules in a flexible price economy where there are cash-in-advance constraints on households' consumption purchases and firms' wage bill. As a result, the competitive equilibrium is subject to a capital accumulation distortion and a portfolio choice distortion. Since these distortions are related to the nominal interest rate, the interest rate peg policy can eliminate them while the money growth peg policy cannot. The former rule therefore is "the benevolent central banker's preferred policy." Monnet and Weber (2001) explains the correlation between money growth and interest rate can be either negative (the liquidity effect) or positive (the Fisher equation effect) and all it "depends on when the change in money occurs and how long the public expects it to last." This explanation applies to monetary policies that are either stated in terms of money supply growth or in terms of interest rates. In an economy with costly trading, Végh (2001) formally establishes conditions for equivalence between three types of monetary policy rules in a continuous-time framework: a "k-percent" money growth rule, a nominal interest rate rule combined with an inflation target, and a real interest rate rule combined with an inflation target. The criterion for equivalence requires that monetary policy rules "yield exactly the same dynamics in response to, say, a long-term reduction in the inflation rate." In a Lucas (1982) cash-in-advance model without capital investment, Schabert (2005) derives equivalent conditions for interest rate and money growth rules by focusing on the restriction that both rules "are consistent with the same fundamental solution to the rational expectations equilibrium." (p.13) Under flexible prices, it is found that a constant money growth rule is equivalent to a passive interest rate rule, while an active interest rate rule behaves like an accommodating money growth rule where money growth responds to the levels of inflation. When prices are sticky, then history dependent interest rate policies are required for equivalence. In addition, constant money growth rules can no longer mimic the Taylor-type interest rate rules. Nevertheless, focusing only on the bubble-free solutions to establish equivalence between interest rate and money growth rules may be too restrictive. As emphasized by Auray and Fève (2002), the equivalent relation can depend "on the relative size of the sunspot variables associated to nominal and real variables" of the dynamic model.

In this paper, we follow the literature to investigate the relation between monetary policy rules that targeting money growth and interest rate. We conduct our analysis in a simple Ak endogenous growth model with flexible prices where a cash-in-advance constraint applies to both consumption and investment goods.[1] By focusing on the equivalence of these policy rules, we require them to yield the same balanced growth path (BGP) equilibria [like in Schabert (2005)] and exhibit qualitatively same equilibrium dynamics [as in Végh (2001)]. Moreover, we add one more criterion, that is, we also restrict the comparative statics results to be qualitatively equivalent [e.g., Wang and Yip (1992) and Zhang (2000)]. The policy rules that we consider allow the central bank to raise its policy target (either the money growth rate or the interest rate) in response to an increase in the rate of inflation. Our main finding is that an active interest rate rule is equivalent to an active money growth rule, where the central bank raises the rate of money growth by more than one percentage point in respond to a one-percentage point increase in inflation. However, a passive money growth rule, where a one-percentage point increase in inflation is associated with a less

[1] The choice of the endogenous growth framework as the analytical vehicle has two merits. First of all, the simple Ak setting allows for analytical solutions. Secondly, money growth rules are rules that determine how the monetary authority would modify the growth rate of real money supply so as to accomodate a change in inflation. For standard neoclassical monetary growth models in a perfect foresight, flexible price environment, prices tend to move almost one for one with the money supply so that it would be difficult to alter the real money supply. With endogenous growth, then a change in the nominal money growth rate does not reflect in an equal-magnitude change in the inflation rate since the equilibrium economic growth rate would also be affected. For a discussion on the endogenous monetary growth models with money growth rules, see Wu and Zhang (1998). For a discussion on the endogenous monetary growth models with interest rate rules, see Itaya and Mino (2004).

than one percentage point increase in the nominal money growth rate, does not mimic a passive interest rate rule. The main reason for the breakdown of policy equivalence under passive monetary rules is the emergence of real indeterminacy in the model. The intuition of having indeterminacy is due to the fact that there is an intertemporal substitution effect that dominates the conventional inflation effect of portfolio allocation. We also find that the magnitude of the intertemporal substitution effect depends on the intertemporal elasticity of substitution in consumption. If the intertemporal elasticity is not too large (say not greater than unity), then it is possible to find examples of policy equivalence under passive rules. For instance, we show that when preferences are logarithmic, then constant money growth rules [Friedman(1959)] can be replicated by an interest rate pegging policy.

The organization of the paper is as follows. The next section provides the basic model for the analysis. Section 3 establishes conditions for the equivalence between money growth and interest rate rules. Concluding remarks are given in section 4.

2 The Model

In this section, we develop the basic monetary endogenous growth model to study the dynamics of equilibrium under money growth rule and interest rate rule. Money is required in advance to purchase consumption and investment goods, or the so-called Stockman (1981) type cash in advance (CIA) constraint.

2.1 The economic environment

Representative Agents. The economy consists of a continuum of identical representative agents with unit mass, each of whom maximizes his lifetime utility according to

$$U = \int_0^{\infty} e^{-\rho t} \frac{c^{1-\sigma} - 1}{1 - \sigma} dt \tag{1}$$

where c is consumption, $\sigma > 0$ is the inverse of the elasticity of intertemporal substitution and $\rho > 0$ is the subjective rate of time preferences.[2] In addition to money the agent can also hold nominal bonds and physical capital. The nominal bonds pay the nominal interest rate $R > 0$. The momentary budget constraint of a typical agent is

$$c + \dot{m} + \dot{b} + \dot{k} = (R - \pi) b + y - \pi m - \tau \tag{2}$$

where $\dot{x} \equiv \frac{dx}{dt}$ is the time derivative of the variable x, k is the capital, b is the real bonds holdings, $m \equiv M/P$ is real money balances defined by deflating the nominal money stock (M) by the price level (P), $\pi \equiv \dot{P}/P$ denotes the inflation rate, τ is the lump-sum transfers and y is the output. The single consumption good prevailing in this economy is produced by a simple Ak technology. Depreciation rate of physical capital is set to zero, a simplification that affects none of our major results.

Following Stockman (1981), each agent faces an additional liquidity constraint given by

$$c + \dot{k} \leq m \tag{3}$$

By defining the agent's non-capital wealth as $a = m + b$, the agent budget constraint can be written as

$$c + \dot{a} + \dot{k} = (R - \pi) a + Ak - Rm - \tau \tag{4}$$

The representative agent's optimization problem is given by maximizing (1) subject to (3), (4), nonnegativity constraints of c, k, M, and the initial asset holdings: $k(0) = k_0 > 0$, $a(0) = a_0 > 0$.

[2] Time index is omitted to ease the burden of notations.

Let ψ be the Lagrangian multiplier associated with the general CIA constraint, λ_k and λ be the costate variables of capital and non-capital wealth respectively. Interior solutions of the above problem are characterized by the first-order conditions:

$$c^{-\sigma} = \lambda + \psi \tag{5}$$

$$\psi = R\lambda \tag{6}$$

$$\lambda + \psi = \lambda_k \tag{7}$$

$$\dot{\lambda} = \lambda \left(\rho + \pi - R\right) \tag{8}$$

$$\dot{\lambda}_k = \rho\lambda_k - \lambda A \tag{9}$$

and the transversality conditions

$$\lim_{t\to\infty} e^{-\rho t}\lambda_k k = \lim_{t\to\infty} e^{-\rho t}\lambda a = 0$$

In addition, the goods market equilibrium condition yields

$$\dot{k} = Ak - c \tag{10}$$

We now perform a balanced growth analysis to solve the for an optimal endogenous monetary growth equilibrium. From $(5), (7)$ and (9), we can solve for

$$\theta = \frac{\dot{c}}{c} = \frac{1}{\sigma}\left(\frac{\lambda}{\lambda_k}A - \rho\right) \tag{11}$$

where θ is the constant growth rate of per capita consumption. By definition, the rate of growth of each endogenous variable (which may not be necessarily equal) is constant along a balanced growth path (BGP). From the goods market equilibrium, we obtain the consumption to capital ratio

$$\frac{c}{k} = A - \frac{\dot{k}}{k}$$

which is constant along a BGP. Thus (per capita) consumption and capital have to grow at same rate, θ. The constancy of the marginal product of capital implies output and capital also have to grow at the same rate. Assuming that the nominal interest rate is positive so that the cash-in-advance constraint always holds with equality, we know that consumption and real money demand grow at the same rate along a BGP. In summary

$$\frac{\dot{k}}{k} = \frac{\dot{c}}{c} = \frac{\dot{m}}{m} = \frac{\dot{y}}{y} = \theta \tag{12}$$

For the costate variable, we can combine (8), (9) and (11) to show that

$$\frac{\dot{\lambda}}{\lambda} = \frac{\dot{\lambda}_k}{\lambda_k} = -\sigma\theta \tag{13}$$

Government. Following Ireland (2003), the Central Bank conducts monetary policy by adjusting a linear combination of the short term nominal interest rate R and the money growth rate μ in response to deviations of inflation from their steady rate values (or targets), according to the policy rule

$$\alpha\left(R - R^*\right) + \beta\left(\mu - \mu*\right) = \gamma_i\left(\pi - \pi^*\right) \text{ and } i = \mu \text{ and } R \tag{14}$$

where α, β and γ_i are the response coefficients chosen by the central bank and R^*, μ^* and π^* are the steady rate (or target) values for R, μ, and π.[3] Since our focus is on the comparison between interest rate targeting and money growth targeting, we consider two special cases.[4] First of all, when $\alpha = 1$, $\beta = 0$ and $\gamma_R \geq 0$, we obtain the standard interest rate rule of Taylor (1993):

$$R = R^* + \gamma_R\left(\pi - \pi^*\right). \tag{15}$$

Following the standard practice, we refer the interest rate policy rule as active if $\gamma_R > 1$ and passive if $\gamma_R < 1$. We further assume $\gamma_R \neq 1$.[5] On the other

[3] The subscript i is used to distinguish different inflation elasticities under interest rate rule and money growth rule

[4] These two types of monetary policy rules are the most commonly adopted rules in the literature.

[5] As in Meng (2002), we assume that fiscal policy is Ricardian so that the present discounted value of total government liabilities converges to zero both in and off equilibrium. For details, see Benhabib et al. (2001) and the reference cited therein.

hand, when $\alpha = 0$, $\beta = 1$ and $\gamma_\mu \geq 0$, it becomes the money growth feedback rule studied by McCallum(1999)

$$\mu = \mu^* + \gamma_\mu (\pi - \pi^*) . \tag{16}$$

Following the practice of interest rate rules, we also refer the money growth rule as active if $\gamma_\mu > 1$ and passive if $\gamma_\mu < 1$. Finally, we like to point out a special case of the money growth rule. When $\gamma_\mu = 0$, we have the constant money growth rule proposed by Milton Friedman (1959) which belongs to the class of passive money growth rules under our classification.

2.2 Equilibrium Analysis

2.2.1 *Money Growth Rules*

When the central bank adopts a money growth rule, R becomes endogenous and equilibrium in money market implies

$$\dot{m} = (\mu - \pi) m \tag{17}$$

We assume the CIA constraint is binding in equilibrium, and when the goods market clear, the binding CIA constraint then implies the quantity of real money holdings exactly equals the quantity of aggregate output. Hence, real money balances and physical capital must be growing at the same rate. Therefore, by considering (10) and (17), we can rewrite the inflation as

$$\pi = \frac{1}{1 - \gamma_\mu} \left(\mu^* - \gamma_\mu \pi^* - A + z \right) \tag{18}$$

where z is the consumption to capital ratio (i.e. $z \equiv c/k$). Then, simple algebra yields

$$\frac{\dot{z}}{z} = \left(\frac{A}{\sigma p} + z - \frac{\rho + \sigma A}{\sigma} \right) \tag{19}$$

where p is ratio of the shadow price of capital to that of non-capital. (i.e. $p \equiv \lambda_k / \lambda$). Using $(6), (7)$ and the definition of p, one can derive the following relationship

$$R = p - 1 \tag{20}$$

Then, we can also derive the following differential equation for p.

$$\frac{\dot{p}}{p} = \left[p - \frac{A}{p} - 1 - \frac{1}{1-\gamma_\mu} \left(\mu^* - \gamma_\mu \pi^* - A + z \right) \right] \tag{21}$$

where we have used (18) and (20) to substitute away the π and R respectively. Then, (19) and (21) constitute the dynamic system that completely characterizes the model's equilibrium under money growth rules.

Comparative Statics Solving (19) and (21), a BGP equilibrium consists of a pair of positive real numbers $(\bar{p}, \bar{z})$ characterized by[6]

$$(\bar{p})^2 - \beta_1 \bar{p} + \beta_2 = 0 \tag{22}$$

and

$$\bar{z} = \frac{\rho}{\sigma} + A \left(1 - \frac{1}{\sigma \bar{p}} \right) \tag{23}$$

where $\beta_1 \equiv \frac{\rho}{\tilde{\sigma}} + 1 + \frac{\mu^* - \gamma_\mu \pi^*}{1-\gamma_\mu}$, $\beta_2 \equiv \left(\frac{1-\tilde{\sigma}}{\tilde{\sigma}} \right) A$ and $\tilde{\sigma} \equiv \sigma \left(1 - \gamma_\mu \right)$.Totally differentiate (22) with respect to $\bar{p}$ and μ^*, we have

$$\frac{d\bar{p}}{d\mu^*} = \frac{\bar{p}}{2 \left(1 - \gamma_\mu \right) \left(\bar{p} - \beta_1/2 \right)}. \tag{24}$$

From (11) and the definition of p, common growth rate is given by

$$\theta = \frac{1}{\sigma} \left[\frac{A}{\bar{p}} - \rho \right]. \tag{25}$$

Hence,

$$\frac{d\theta}{d\mu^*} = - \frac{A}{\sigma (\bar{p})^2} \frac{d\bar{p}}{d\mu^*} = - \frac{A}{2\sigma \mu^* \bar{p} \left(1 - \gamma_\mu \right) \left(\bar{p} - \beta_1/2 \right)}. \tag{26}$$

Solving the quadratic equation (22), we get the roots:

$$\bar{p} = \frac{\beta_1 \pm \sqrt{\Delta}}{2}, \tag{27}$$

[6] An overhead bar denotes the balanced growth path value of a variable.

where $\Delta \equiv (\beta_1)^2 - 4\left(\frac{1-\tilde{\sigma}}{\tilde{\sigma}}\right) A$.[7] When $\tilde{\sigma} < 0$ or $\tilde{\sigma} > 1$, one of the roots is negative and has to be rejected due to the nonnegativity restriction of $\bar{p}$. In this case, a unique BGP equilibrium exists with $\bar{p} > \beta_1/2$. Hence, according to (26), an increase in the growth rate of nominal money supply will reduce (increase) the long-run common growth rate of other aggregates when $\tilde{\sigma} > 1 (< 0)$. Figures 1a and 1b provide the graphical representation of our comparative statics result for the case where $\tilde{\sigma} > 1$ and $\tilde{\sigma} < 0$ respectively.

[INSERT FIGURE 1a AND 1b HERE]

From (19) and (21), it is straightforward to show that the equilibrium locus $\dot{z} = 0$ is upward sloping. On the other hand, the $\dot{p} = 0$ locus is upward (downward) sloping when $\tilde{\sigma} > 1 (< 0)$. Specifically, when it is upward sloping, the $\dot{p} = 0$ locus should have a steeper slope than the $\dot{z} = 0$ locus. An increase in nominal money growth shifts the $\dot{p} = 0$ locus to the left so that $d\bar{p}/d\mu > 0 (< 0)$ for $\tilde{\sigma} > 1 (< 0)$.

On the other hand, when $0 < \tilde{\sigma} < 1$ so that the intertemporal elasticity of substitution in consumption must be larger than unity ($\sigma < 1$) and nominal money growth cannot adjust more than the inflation rate ($1 > \gamma_\mu > 0$), two possible values of $\bar{p}$ emerge as shown in Figure 2.

[INSERT FIGURE 2 HERE]

This is due to the fact that the relative magnitudes of the slopes of the two equilibrium loci depend on the values of $\bar{p}$. Let $\bar{p}_1$ and $\bar{p}_2$ denote the two roots

[7] To ensure the existence of real root(s), we assume that $\Delta \geq 0$.

with $\bar{p}_1 > \bar{p}_2$.[8] It is then straightforward to derive the following results

$$\bar{p}_1 > \frac{\beta_1}{2} > \bar{p}_2, \quad \theta(\bar{p}_2) > \theta(p_1) \quad \text{and} \quad \frac{d\theta(\bar{p}_2)}{d\mu^*} > 0 > \frac{d\theta(\bar{p}_1)}{d\mu^*}.$$

The intuition of the above results can be obtained from (18) which indicates that a change in the money growth target affects both the inflation rate (π) and the consumption-capital ratio (z). First, we consider the effect on π which we regard it as the conventional inflation effect. According to (21), we have

$$\bar{\pi} = \bar{p} - \frac{A}{\bar{p}} - 1 \tag{28}$$

so that $\bar{\pi}$ and $\bar{p}$ are directly related at the BGP equilibrium. Such a positive correlation reflects the no-arbitrage condition in the portfolio. Consider a rise in the rate of inflation which lowers the return for real money balances and hence leads to a portfolio substitution from real balances to capital. This increase in the demand for capital bids up its relative price (i.e., a rise in $\bar{p}$), reduces its real return, and in turn retards economic growth. Notice that the effect of money growth targeting on inflation depends on the feedback parameter γ_μ. If money growth responds more than proportionately to inflation changes, then the inflation effect becomes negative as the stock of real balances shrinks over time. Next, we consider a less conventional effect of money growth on economic growth through the consumption-capital ratio (z) which we call the intertemporal substitution effect. According to (18), other things being equal, a rise in the money growth target lowers the consumption-capital ratio. This requires an intertemporal substitution from consumption to investment so that capital stock is increased. The expansion in the availability of capital stock lowers its price as indicated in (23). When $\bar{p}$ falls, economic growth is promoted. Notice

[8] In order to ensure that both roots satisfy the transversality conditions, an upper bound on the nominal money growth rate given by

$$\frac{(1-\tilde{\sigma})A}{\rho} > 1 + \mu^*$$

is needed when $0 < \tilde{\sigma} < 1$.

that the magnitude of this intertemporal substitution effect is proportional to the intertemporal elasticity of substitution in consumption $(1/\sigma)$. Specifically, if the intertemporal elasticity is large (greater than unity) or σ is small (below unity), then this intertemporal substitution effect is strong enough to dominate the conventional inflation effect so that higher money growth leads to faster economic growth (i.e., $d\theta/d\mu > 0$). For the unique BGP equilibrium case where $\tilde{\sigma} < 0$ (or $\gamma_\mu > 1$), both the conventional inflation and intertemporal substitution effects work in the same direction so that a rise in money growth target accelerates economic growth. When $\tilde{\sigma} > 1$ (hence $\sigma > 1$), the two effects work in opposite direction and the conventional inflation effect dominates so that faster money growth retards economic growth. Finally, in order to understand the comparative statics under multiple BGP equilibria, we note that the final outcome of the results depends on the magnitude of $\bar{p}$. According to (21), the effect of inflation on $\bar{p}$ depends on the original equilibrium value of p :

$$\frac{1}{1-\gamma_\mu} d\mu^* = \left(1 - \frac{1-\tilde{\sigma}}{\tilde{\sigma}} \frac{A}{\bar{p}^2}\right) d\bar{p}.$$

So the larger $\bar{p}$ we have, the more likely that the conventional inflation effect dominates the intertemporal substitution effect on growth. This explains why for the case where $0 < \tilde{\sigma} < 1$, we obtain the conventional result that faster money growth suppresses economic growth at the low-growth (or high $\bar{p}$) equilibrium.

Local Dynamics Linearizing the system of (19) and (21) in the neighborhood of a BGP:

$$\begin{bmatrix} \dot{p} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} \bar{p}\left[1 + \dfrac{A}{(\bar{p})^2}\right] & -\dfrac{\bar{p}}{1-\gamma_\mu} \\ -\dfrac{A}{\sigma(\bar{p})^2}\bar{z} & \bar{z} \end{bmatrix} \begin{bmatrix} p - \bar{p} \\ z - \bar{z} \end{bmatrix}. \tag{29}$$

The determinant and trace of the Jacobian matrix J are

$$\det J = \bar{p}\bar{z}\left[1 - \frac{A}{(\bar{p})^2}\left(\frac{1-\tilde{\sigma}}{\tilde{\sigma}}\right)\right] \tag{30}$$

$$Trace\ J = \bar{p}\left[1 + \frac{A}{(\bar{p})^2}\right] + \bar{z} > 0. \tag{31}$$

To determine the sign of $\det J$, we consider $\dot{p} = 0$ at equilibrium. From (22), we have

$$\det J = 2\bar{z}\left(\bar{p} - \beta_1/2\right).$$

It is clear that $\det J$ must be positive when $\tilde{\sigma} > 1$, since the fraction inside the bracket must be smaller than one by the fact that $\bar{p} \geq 0$. By inspecting (30), $\det J$ is also positive when $\tilde{\sigma} < 0$. The BGP equilibrium in these two cases is therefore a source so that local indeterminacy cannot occur. But when $0 < \tilde{\sigma} < 1$, $\det J > 0$ (< 0) for the BGP with $\bar{p}_1$ $(\bar{p}_2)$ (where $\bar{p}_1 > \bar{p}_2$). Thus the low- (high-) growth equilibrium is a source (saddle) in this subcase. As a result, local indeterminacy occurs at the high-growth BGP equilibrium. This finding is consistent with the existing literature that in order to have indeterminacy in one-sector models the intertemporal elasticity of substitution in consumption cannot be too low.

Proposition 1 *In the Ak model with feedback money growth rule, we have non-superneutrality in the growth-rate sense. When $\tilde{\sigma} > 1$ or $\tilde{\sigma} < 0$ there exists a unique balanced growth path equilibrium that is locally determinate and faster money growth retards (promotes) economic growth in the former (latter) case. When $0 < \tilde{\sigma} < 1$, then dual BGP equilibria may emerge in which the low-growth equilibrium is locally determinate where faster money growth lowers economic growth, and the high-growth equilibrium is indeterminate where faster money growth raises economic growth.*

To understand the relation between indeterminacy and intertemporal substitution, we consider an initial decline in the relative price of capital (p) from its BGP level under the assumption that the intertemporal substitution effect from consumption to investment dominates. Such a drop in p then raises the real return to capital as well as its accumulation relative to consumption due to intertemporal substitution. Thus, the consumption-capital ratio (z) falls so that p begins to rise according to its law of motion equation (21). As a result, the initial reduction of p is reversed and the trajectory considered is consistent with an equilibrium in which p converges to $\bar{p}$. The intertemporal substitution effect then engineers real indeterminacy in the model. On the other hand, if the inflation effect on portfolio adjustment dominates, then local indeterminacy cannot occur. In this case, the no-arbitrage condition requires a drop in the inflation rate to match the initial decline in p. By portfolio substitution from capital to real money balances, the consumption-capital ratio (z) rises accordingly. From (21), this leads to a further decline in p so that the resulting trajectory would not be consistent with an equilibrium in which p converges to $\bar{p}$.

2.2.2 *Interest Rate Rules*

When the central bank choose an interest rate rule, m becomes endogenous. Combining (20) and (15), we can now rewrite the inflation as

$$\begin{aligned} \gamma_R(\pi - \pi^*) + R^* &= p - 1 \qquad (32) \\ \pi &= \frac{1}{\gamma_R}(p - 1 - R^*) + \pi^* \end{aligned}$$

which is the counterpart to (18). Next, we use (32) and (20) to substitute away the π and R again to derive the following differential equation in p

$$\begin{aligned} \frac{\dot{p}}{p} &= \left[p - \frac{A}{p} - 1 - \frac{1}{\gamma_R}(p - 1 - R^*) - \pi^*\right] \qquad (33) \\ &= \left[p - \frac{A}{p} - \frac{p}{\gamma_R} + \Omega\right] \end{aligned}$$

where $\Omega \equiv \dfrac{R^* + 1 - \gamma_R(1 + \pi^*)}{\gamma_R}$.

Comparative Statics Solving (19) and (33), a BGP equilibrium consists of a pair of positive real numbers $(\bar{p}, \bar{z})$ characterized by

$$\beta_3 (\bar{p})^2 + \Omega \bar{p} - A = 0 \tag{34}$$

and

$$\bar{z} = \frac{\rho}{\sigma} + A\left(1 - \frac{1}{\sigma \bar{p}}\right) \tag{35}$$

where $\beta_3 = \left(1 - \frac{1}{\gamma_R}\right)$. Solving the quadratic equation (34) , we get the roots

$$\bar{p} = \frac{-\Omega \pm \sqrt{\Delta}}{2\beta_3}, \tag{36}$$

where $\Delta \equiv \Omega^2 + 4\left(1 - \frac{1}{\gamma_R}\right) A > 0$.[9] When interest rate rules are active so that $\gamma_R > 1$, one of the roots is negative and has to be rejected due to the nonnegativity restriction of $\bar{p}$. In this case, a unique BGP equilibrium exists with $\bar{p} = \left(-\Omega + \sqrt{\Delta}\right)/2\beta_3$. On the other hand, when we have passive interest rate rules ($\gamma_R < 1$), two possible values of $\bar{p}$ emerge. Let $\bar{p}_1$ and $\bar{p}_2$ denote the two roots with $\bar{p}_1 > \bar{p}_2$.

Totally differentiate (11) with respect to R^*,

$$\frac{d\theta}{dR^*} = -\frac{A}{\sigma (\bar{p})^2} \frac{d\bar{p}}{dR^*} \tag{37}$$

The following lemma is useful to determine the sign of $\frac{d\bar{p}}{dR^*}$under active and passive interest rate rules.

Lemma 2 $|\Omega| > (<) \sqrt{\Delta}$ *for passive (active) interest rate rules.*

Proof. *From the definition of* Δ *and notice that* $1 - \frac{1}{\gamma_R} > (<) \ 0$ *for the case of active (passive) interest rate rules, and the result follows.* ■

For any active interest rate rule, the only positive root is $\bar{p} = \left(-\Omega + \sqrt{\Delta}\right)/2\beta_3$ so that simple differentiation yields

$$\frac{d\bar{p}}{dR^*} = \frac{\beta_3 - 1}{2\beta_3}\left[1 - \frac{\Omega}{\sqrt{\Delta}}\right] < 0 \text{ and } \frac{d\theta}{dR^*} > 0$$

[9] By taking $\Delta > 0$, we only focus on the real root solutions.

Therefore, an increase in R^* will increase the growth rate and the corresponding figure is provided in figure 3.

[INSERT FIGURE 3 HERE]

In this case, a rise in the interest rate target lowers the shadow relative price of capital. This in turn fasten economic growth through more rapid capital accumulation. On the other hand, for any passive interest rate rule, we have

$$\frac{d\bar{p}_1}{dR^*} = \frac{\beta_3 - 1}{2\beta_3}\left[1 + \frac{\Omega}{\sqrt{\Delta}}\right] > 0 > \frac{d\bar{p}_2}{dR^*} = \frac{\beta_3 - 1}{2\beta_3}\left[1 - \frac{\Omega}{\sqrt{\Delta}}\right]$$

and hence

$$\frac{d\theta\left(\bar{p}_2\right)}{dR^*} > 0 > \frac{d\theta\left(\bar{p}_1\right)}{dR^*}$$

where $\bar{p}_1 > \bar{p}_2$ and the corresponding figure is provided in figure 4.

[INSERT FIGURE 4 HERE]

To explain the intuition behind these comparative statics results, we need to identify the two underlying forces at work for an increase in the interest rate target. From (15), we have

$$R^* = R - \gamma_R \pi + \gamma_R \pi^*$$

so that a change in the interest rate target R^* affects both the interest rate and the inflation rate. Consider a rise in the interest rate target. This leads to an increase in the interest rate so that the return to capital rises for a given rate of inflation. As a result, an intertemporal substitution from consumption to capital investment takes place and the increase in the demand for capital raises its shadow price. As before, we denote it as the intertemporal substitution effect. On the other hand, the contractionary nature of the increase in R^* suppresses inflation so that the real return to money rises. This in turn leads to a portfolio reallocation from capital to money and hence a fall in the shadow price of capital. This is the conventional inflation effect of interest rate targeting. On net, the effect on $\bar{p}$ can be computed from (33):

$$dR^* = \left[1 - \gamma_R \left(1 + \frac{A}{\bar{p}^2}\right)\right] d\bar{p}$$

where the first (second) term in the square bracket of the RHS represents the intertemporal substitution (inflation) effect. Notice that the inflation effect depends on the initial equilibrium level of p. Under active interest rate rules so that $\gamma_R > 1$, the inflation effect dominates so that an increase in the interest rate target lowers $\bar{p}$ and promotes growth. However, under passive rules when $\gamma_R < 1$, the net effect is in general ambiguous and it depends on the initial equilibrium level of p. Thus, at the high- (low-) growth equilibrium, the inflation effect dominates (is dominated by) the intertemporal substitution effect so that an increase in the interest rate target reduces (raises) the shadow relative price of capital. This in turn accelerates (retards) economic growth through faster (slower) capital accumulation.

Local Dynamics Because of the block recursive nature, the dynamics under interest rate rule can be completely described by the differential equation in p only. We linearize (33) around the BGP to yield

$$\dot{p} = \eta \left(p - \bar{p}\right)$$

where $\eta \equiv \bar{p}\left[\left(1 - \frac{1}{\gamma_R}\right) + \frac{A}{(\bar{p})^2}\right]$ is the eigenvalue to the differential equation (33). For active interest rate rule $\gamma_R > 1$, it is easy to show that $\eta > 0$ and hence the dynamics is determinate. To further investigate the case of passive interest rate rule, we use (34) and (36) to simplify η and we have the following lemma

Lemma 3 *$sgn\left\{\eta\right\} = sgn\left\{\frac{2A}{\Omega} - \bar{p}\right\} = sgn\left\{\frac{\sqrt{\Delta}\left[\sqrt{\Delta} \pm \Omega\right]}{2\Omega\left(1 - \frac{1}{\gamma_R}\right)}\right\}$.*

Proof. *We first note that* η *can be written as* $\left(\frac{\Omega}{\bar{p}}\right)\left(\frac{2A}{\Omega}-\bar{p}\right)$ *by (34). Thus, we obtain the first equality. To obtain the second equality, we use (36) to substitute away* $\bar{p}$ *and yield* $\frac{\Delta \pm \Omega\sqrt{\Delta}}{2\Omega\left(1-\frac{1}{\gamma_R}\right)}$*. Factorize* $\sqrt{\Delta}$ *out gives the result.* ■

Next, first notice that the non-negativity constraint of $\bar{p}$ implies that $\Omega > 0$ so that we have $\Omega > \sqrt{\Delta}$ under passive interest rate rules ($\gamma_R < 1$). This together with Lemma 2 above then yield $\eta < (>) 0$ for the low- (high-) growth equilibrium $\bar{p}_1 (\bar{p}_2)$. Our findings that active interest rate rules yield determinacy while indeterminacy is possible under passive rules are consistent with the conventional conclusion in the literature.[10] We now summarize our characterization of the dynamics for interest rate rules in the following proposition:

Proposition 4 *In the Ak model with active interest rate rule* ($\gamma_R > 1$)*, there is a unique BGP equilibrium and it is locally determinate. When interest rate rule becomes passive* ($\gamma_R < 1$)*, dual BGP equilibria emerge. The high-growth equilibrium* ($\bar{p}_2$) *is locally determinate while the low-growth equilibrium* ($\bar{p}_1$) *is indeterminate. Also, at the high- (low-) growth equilibrium, an increase in the interest rate target accelerates (retards) economic growth.*

The explanation for the possibility of obtaining indeterminacy under passive interest rate rules can be given as follows. Recalling (7) and (9), the law of motion of the shadow price of capital p becomes

$$\frac{\dot{p}}{p} = \left[(R-\pi) - \frac{A}{p}\right]. \tag{38}$$

Consider an increase in p from its initial BGP equilibrium level. The direct impact is that p begins to rise since (38) implies that $\dot{p}/p > 0$. The magnitude of this direct intertemporal substitution effect is inversely related to the initial equilibrium level of p. Next, according to (20), the increase in p also raises the

[10] For a discussion on the conventional findings of interest rate rules, see Meng and Yip (2004) and Yip and Li (2004).

nominal rate of interest R. Under an active interest rate policy, the real rate of interest also rises which then leads to a further increase in p from (38) due to a portfolio reallocation from money to capital. As a result, such a trajectory of p is not consistent with a BGP equilibrium. On the other hand, if the interest rate rule is instead passive, then the real interest rate falls so that we have a drop in p as $\dot{p}/p < 0$. If the initial $\bar{p}$ is large enough, then the negative intertemporal substitution effect of the real interest rate on p dominates the positive inflation effect of portfolio allocation. Thus the trajectory under construction is consistent with the BGP equilibrium so that indeterminacy occurs. Once again, the dominance of the intertemporal substitution effect is the key to generate indeterminacy in the model.

3 Equivalence on Monetary Policies

To establish equivalence between money growth rules and interest rate rules, we adopt all the criteria that are commonly used in the literature. Specifically, we require the monetary policy rules under consideration to be identical both at the BGP equilibrium as well as along the transition paths. We first provide our definition of policy equivalence as follows:

Definition 5 *Two monetary policy regimes are equivalent if*

1. both policy rules yield the same BGP equilibria, and

2. both the BGP equilibria exhibit same equilibrium dynamics, and

3. the comparative statics results are qualitatively equivalent at the determinate BGP equilibrium.

Recall first the quadratic equations (22) and (34) which are now reproduced here for convenience. For the case of feedback money growth rules, we have

$$(\bar{p})^2 - \left(1 + \frac{\rho}{\tilde{\sigma}} + \frac{\mu^* - \gamma_\mu \pi^*}{1 - \gamma_\mu}\right)\bar{p} + \left(\frac{1 - \tilde{\sigma}}{\tilde{\sigma}}\right)A = 0.$$

For interest rate rules, we have

$$(\bar{p})^2 - \left(1 + \frac{R^*}{1-\gamma_R} - \frac{\gamma_R \pi^*}{1-\gamma_R}\right)\bar{p} + \left(\frac{\gamma_R}{1-\gamma_R}\right)A = 0.$$

For the two quadratic equations to yield identical BGP equilibrium solutions, we can impose the following conditions

$$\frac{1-\tilde{\sigma}}{\tilde{\sigma}} = \frac{\gamma_R}{1-\gamma_R}, \tag{39}$$

$$\frac{\rho}{\tilde{\sigma}} + \frac{\mu^* - \gamma_\mu \pi^*}{1-\gamma_\mu} = \frac{R^* - \gamma_R \pi^*}{1-\gamma_R}. \tag{40}$$

Rearranging (39), we have

$$\gamma_R = 1 - \sigma\left(1 - \gamma_\mu\right) = 1 - \tilde{\sigma}. \tag{41}$$

Since $\gamma_R > 0$, we must have $\tilde{\sigma} < 1$. According to Proposition 1, when $\tilde{\sigma} < 0$, a rise in money growth accelerates economic growth.[11] Next, substituting (41) into (40) and assuming the inflation target remains unchanged regardless of the types of monetary policy rules being practiced, we obtain

$$R^* = \sigma\mu^* + [\rho + (1-\sigma)\pi^*]. \tag{42}$$

We first note that (42) implies that the interest-rate and money-growth targets are positively related in policy design, revealing the long-run "Fisher equation view" emphasized by Monnet and Weber (2001). We assume the exogenous target can be manipulated so that (40) always hold, immediate results can then be inferred from (41), a passive interest rule $\gamma_R < 1$ and a passive money growth rule with $\gamma_\mu < 1$ can lead to identical BGP equilibrium solution. Notice that constant money rule ($\gamma_\mu = 0$) is a subcase in this situation. Similarly, active interest rule $\gamma_R > 1$ and active money growth rule $\gamma_\mu > 1$ can lead to identical

[11] Thus, the possibility of getting a unique BGP equilibrium under money growth rules with $\tilde{\sigma} > 1$ is ruled out by our equivalent criteria. In addition, in practice, it is widely believed that the intertemporal elasticity of substitution in consumption cannot be larger than unity ($\sigma \geq 1$), then $\gamma_R > 0$ implies that the case where $\gamma_\mu < 0$ must be ruled out.

BGP equilibrium solutions. We also like to point out that (42) implies that the comparative statics of R^* and μ^* are positively correlated. This is important for establishing qualitative equivalent comparative statics results in later analysis.

We next investigate equivalence for the equilibrium dynamics between the monetary policy rules. For active interest rate rules, the BGP equilibrium is unique and it is locally determinate. Under money growth rules, when $\tilde{\sigma} < 0$, the BGP equilibrium exhibits the same dynamic properties. For the comparative statics, we have shown that

$$sgn(\frac{d\bar{p}}{d\mu^*}) = sgn(\frac{d\bar{p}}{dR^*}).$$

Thus we can claim that active interest rate rules is equivalent to an active money growth rule with $\tilde{\sigma} < 0$ (or $\gamma_\mu > 1$).

For both passive interest rate rules and passive money growth rules ($0 < \tilde{\sigma} < 1$ or $1 > \gamma_\mu > 1 - 1/\sigma$), dual BGP equilibria emerge and in each case one of the BGP equilibria is locally indeterminate while the other determinate. In addition, an increase in the target money growth rate (μ^*) and an increase in the target nominal interest rate (R^*) both produce the same comparative statics results qualitatively:

$$\frac{d\bar{p}_2}{di^*} < 0 < \frac{d\bar{p}_1}{di^*},$$

where $\bar{p}_1 > \bar{p}_2$ and $i = \mu, R$. Table 1 provides a summary for the comparative statics results.

[INSERT TABLE 1 HERE]

However, the low-growth equilibrium under passive money growth rules is locally indeterminate while it is determinate under passive interest rate rules. As a result, the comparative statics at the determinate BGP equilibrium are reversed between the two passive monetary policy rules under consideration. If we focus only on the bubble-free or determinate equilibrium as in Schabert (2005), then the policy implications derived from the comparative statics are completely different between the two types of passive monetary policy rules.

We now summarize our main findings on equivalence in the following proposition:

Proposition 6 *Consider the following two types of monetary policy feedback rules:*

1. money growth rules: $\mu = \mu^* + \gamma_\mu (\pi - \pi^*)$,

2. interest rate rules: $R = R^* + \gamma_R (\pi - \pi^*)$.

Then an active interest rate rule ($\gamma_R > 1$*) is equivalent to an active money growth rule (*$\gamma_\mu > 1$*) where a unique determinate BGP equilibrium emerges and an increase in the monetary policy target improves economic growth performance. On the other hand, under passive interest rate rules (*$\gamma_R < 1$*) and passive money growth rules (*$1 > \gamma_\mu > 1 - 1/\sigma$*), real indeterminacy can occur. Also, we are unable to establish equivalence between passive monetary policy rules because the comparative statics results at the determinate BGP equilibrium are not qualitatively equivalent.*

When monetary policy rules are active, then the inflation effect of portfolio substitution dominates the intertemporal interest rate effect so that a rise in the policy target pushes up the rate of economic growth. In addition the "more-than-proportionate" responses of active monetary targets to changes in inflation is destabilizing in nature so that the BGP equilibrium is determinate. On the other hand, when the destabilizing force is absent under passive monetary rules and the intertemporal substitution effect dominates, then indeterminacy can occur. In the presence of real indeterminacy, policy implications derived from the comparative statics results can be very different so that it is difficult to establish equivalence between the policy rules.

Before closing the section, we would like to consider a couple of interesting special cases for equivalence analysis. According to the above proposition, the possibility of indeterminacy breaks the policy equivalence between passive money growth rules and passive interest rate rules. As constant money growth rule is a special type of passive money growth rules ($1 > \gamma_\mu = 0$), we may expect that it is not equivalent to an interest rate rule. However, strictly speaking, our conjecture is correct when the intertemporal elasticity of substitution in consumption is greater than unity. In this case, we have $1 - 1/\sigma < 0$ so that the range of γ_μ being considered under passive money growth rules covers also the constant money growth case (i.e., $1 > \gamma_\mu = 0 > 1 - 1/\sigma$).

Corollary 7 *If the intertemporal elasticity of substitution in consumption is greater than unity ($\sigma < 1$), constant money growth rules cannot mimic any feedback interest rate rules.*

However, there is an exception to the above non-equivalence finding of constant money growth rules. Consider the special case of the interest rate pegging, i.e., $\beta = \gamma_R = 0$, so that $R = R^*$ according to (15). Using the first-order conditions (6), (7) and the definition of p, we can show that p is always constant:

$$p = 1 + R^*. \tag{43}$$

As a result, we have $\dot{p}/p = 0$ for all time so that π is always constant:

$$\pi^* = R^* - \frac{A}{1 + R^*}. \tag{44}$$

To establish equivalence with money growth rules, we follow (41) and (42) to derive[12]

$$\gamma_\mu = 1 - 1/\sigma, \tag{45}$$

[12] Schabert (2005) also obtains the equivalent condition (45) and claims that an interest rate peg is equivalent to a money growth rule satisfying $\gamma_\mu = 1 - 1/\sigma$, "therefore implies money supply to be accommodating, i.e., to be positively related to inflation, if $\sigma > 1$." (p.16) However, as shown in the above analysis, the dynamics associated with interest rate pegging is very different from that of a pssive money growth rule so that equivalence cannot be established under our criteria.

$$\sigma R^* + \frac{(1-\sigma)A}{1+R^*} = \rho + \sigma\mu^*. \tag{46}$$

To sharpen our focus, we consider the special case where preferences are logarithmic (i.e., $\sigma = 1$). Then (45) implies that $\gamma_\mu = 0$ and we have constant money growth rules. The BGP equilibrium solution of p is

$$\bar{p} = 1 + R^* = 1 + \rho + \mu^*, \tag{47}$$

so that the comparative statics results for both monetary policy rules are identical:

$$d\bar{p}/dR^* = d\bar{p}/d\mu^* = 1.$$

For equilibrium dynamics, it can be easily shown that the BGP equilibrium is a unique source locally under constant money growth rules. Thus equivalence can be established between these two types of pegging rules with logarithmic preferences.

Proposition 8 *When the felicity function is logarithmic ($\sigma = 1$), then nominal interest rate pegging policies ($\gamma_R = 0$) are equivalent to constant money growth rules ($\gamma_\mu = 0$).*

The intuition of the exception that interest rate pegging is equivalent to money growth pegging under logarithmic preferences is not difficult to understand. We first recall that the presence of real indeterminacy under passive monetary policy rules is due to the fact that the intertemporal substitution effect dominates the conventional inflation effect of portfolio allocation. But the dominance of the intertemporal substitution effect requires the intertemporal elasticity of substitution in consumption to be sufficiently larger (in general greater than unity). With logarithmic preferences, then the intertemporal elasticity of substitution in consumption is restricted to be unity so that the portfolio allocation effect of inflation dominates and real indeterminacy cannot occur. By removing the possibility of indeterminacy, we are able to restore an equivalent relation between these special passive monetary policy rules.

4 Concluding Remarks

In a simple Ak endogenous growth model with flexible prices where a cash-in-advance constraint applies to both consumption and investment goods, we have investigated the equivalent relation between money growth and interest rate rules. We have considered general money growth rules that allow the money growth rate to depend on the rate of inflation. In the analysis, we have restricted these monetary policy rules to yield the same balanced growth path equilibria, to exhibit the same equilibrium dynamics, and to have qualitatively equivalent comparative statics results. Our main finding is that an active interest rate is equivalent to an active money growth rule, where the central bank raises its monetary policy target by more than one percentage point in respond to a one-percentage point increase in inflation. However, under passive monetary policy rules, equivalence cannot be established due to the possibility of real indeterminacy. An exception can be found when preferences are logarithmic (the intertemporal elasticity of substitution in consumption is unity), then constant money growth rules [Friedman(1959)] mimic interest rate pegging rules.

To close the paper, we would like to suggest an interesting extension of our analysis by turning to the case where prices are no longer flexible. When prices are sticky, the conditions for equivalence between money growth and interest rate rules can be very different from the case of flexible prices. Specifically, real money balances are now sluggish so that the dimension of the state variable system is augmented. As a result, the fundamental solution of the model is history dependent for a money growth regime, but not under an interest rate rule. We plan to study this extension in an accompanying paper.

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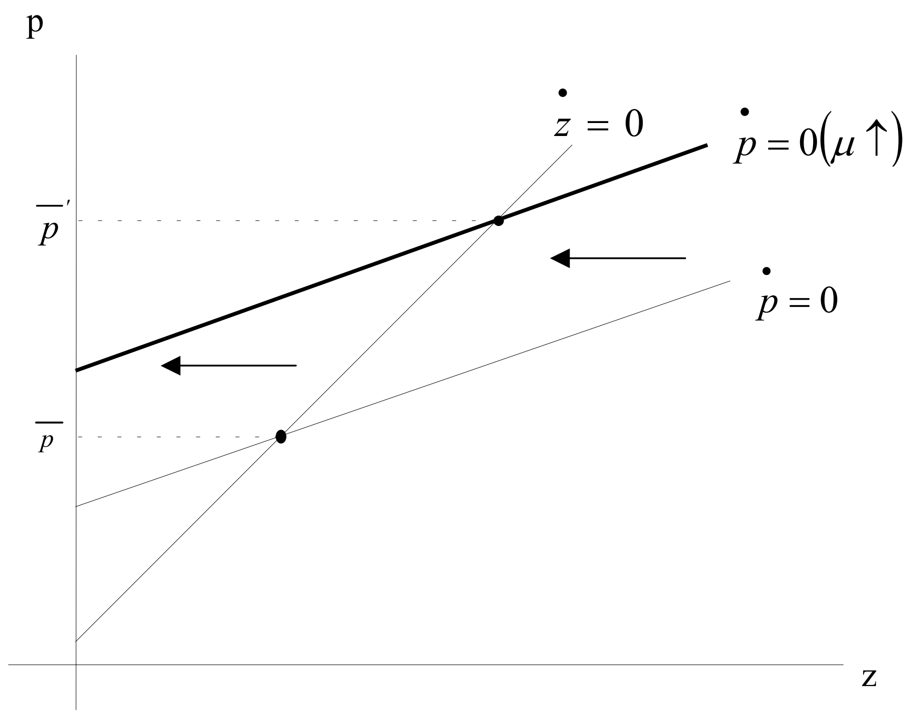

Figure 1a Comparative Statics for an increase in μ* when $\tilde{\sigma} > 1$

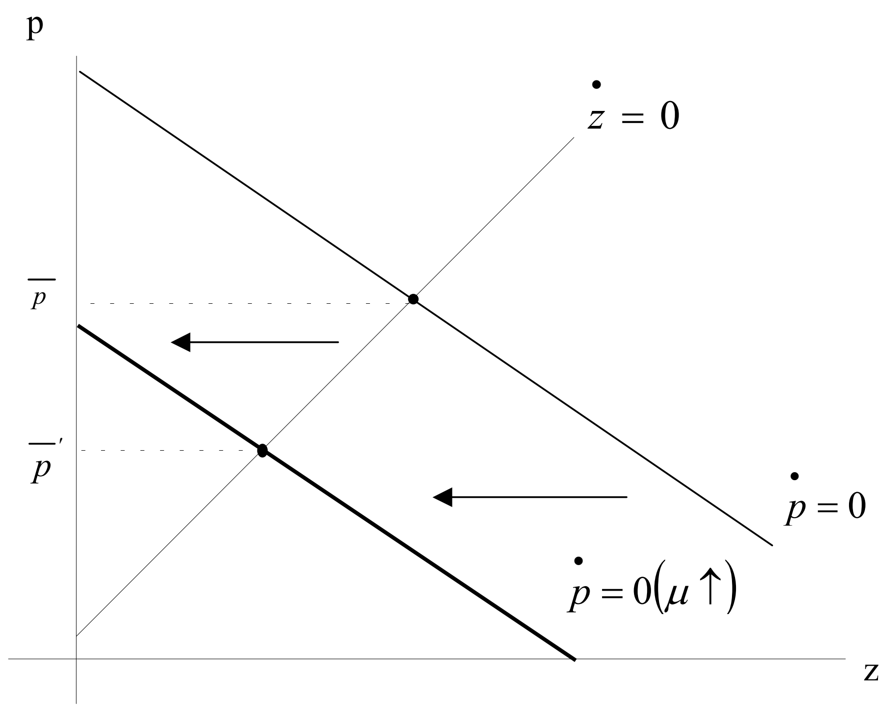

Figure 1b Comparative Statics for an increase in μ* when $\tilde{\sigma} < 0$

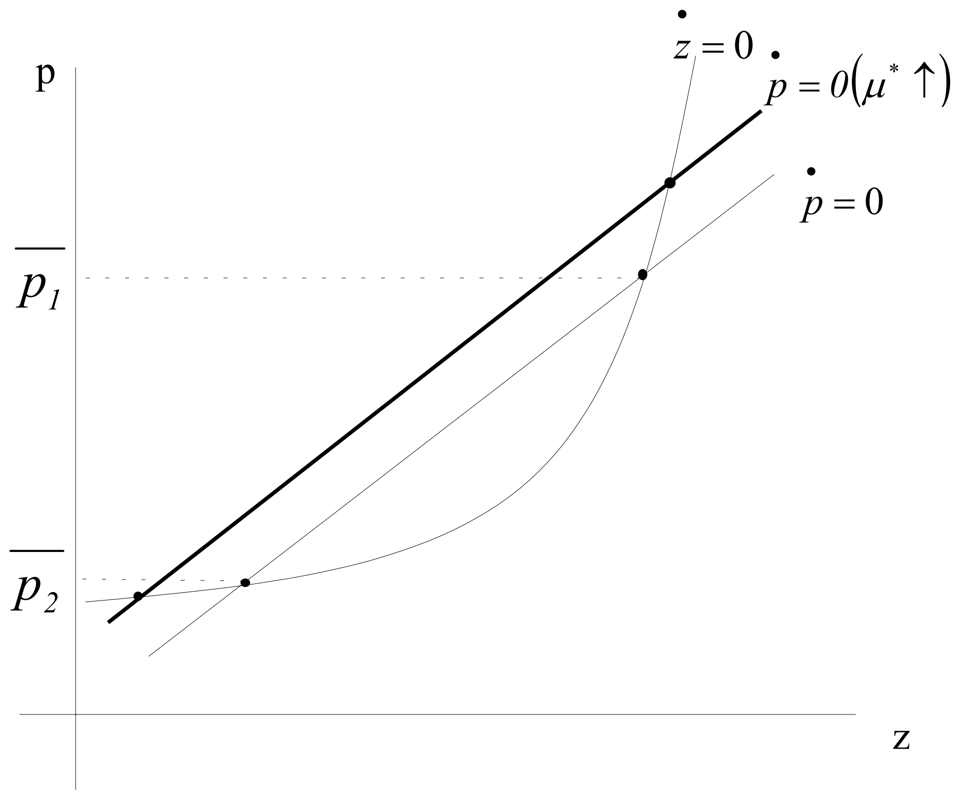

Figure 2 Comparative Statics for an increase in μ* when $0 < \tilde{\sigma} < 1$

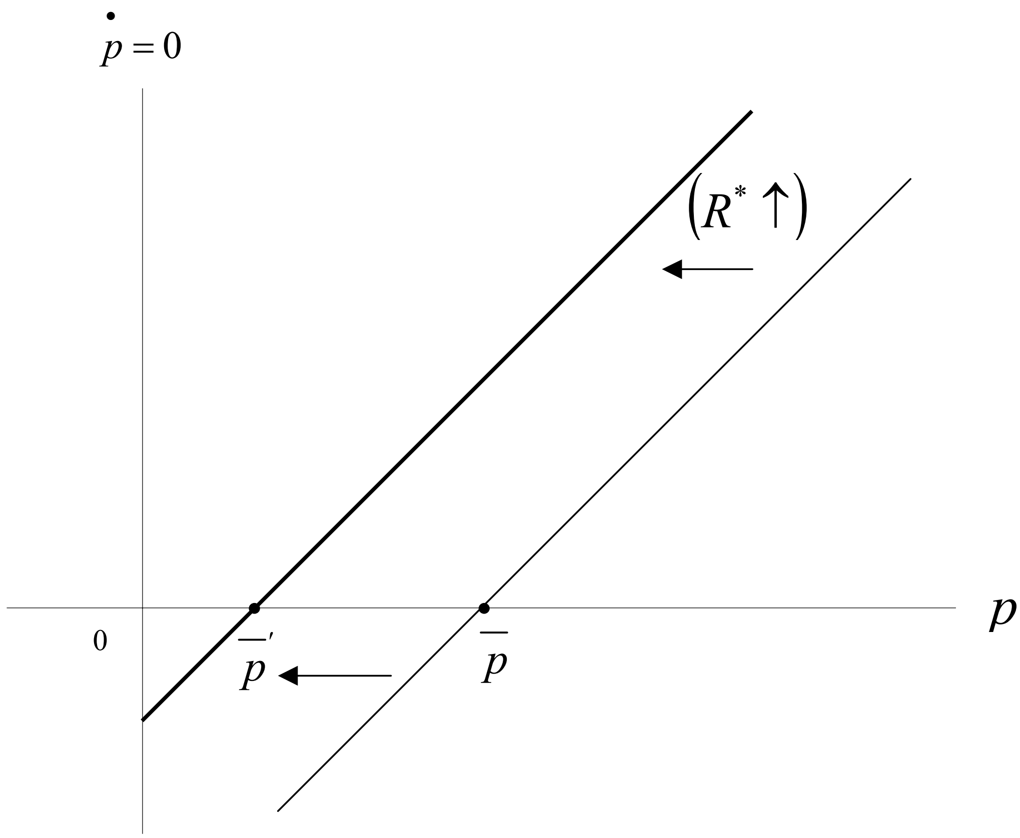

Figure 3 Comparative Statics for an increase in *R** under active interest rate rule

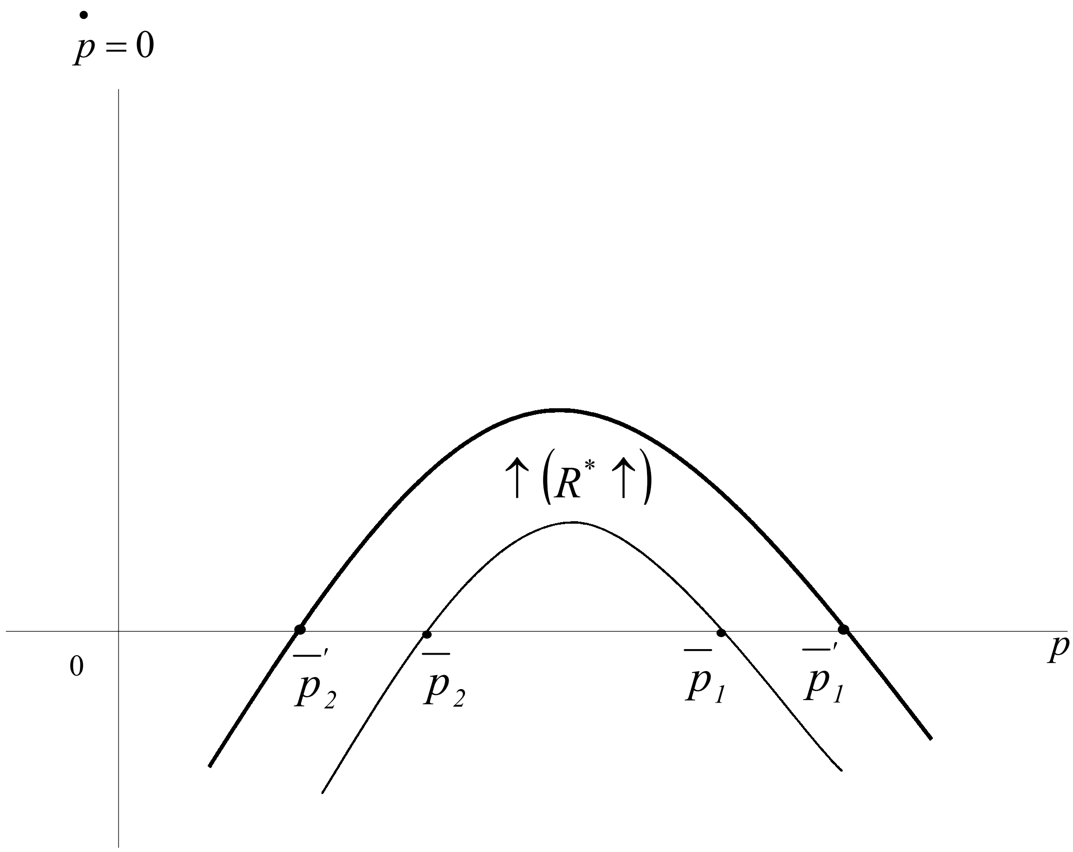

Figure 4 Comparative Statics for an increase in *R** under passive interest rate rule