A Theory of Organizational Dynamics: Internal Politics and Efficiency*

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Abstract

In this paper, we develop a stylized model to study how internal politics affects an organization's hiring of new members and investigate the implications of the dynamic interactions between internal politics and hiring of new members on the organization's long run outcomes and welfare. We consider an organization with a fixed size in which one of the incumbent members retires in each period and the incumbent members vote to admit a candidate to fill the vacancy. Agents differ by quality that is valued equally by every member in the organization. In addition, each agent belongs to one of the two types, where members of the majority type in any period control the rent distribution of the organization and share the total rent of that period among themselves. We characterize the conditions for a Markov equilibrium of the dynamic game, and the long run equilibrium outcome and welfare. Then we solve the model of a three member club with uniformly distributed quality, under both majority and unanimity voting rule in admitting new members. Among other things, we find that "some politics can be a good thing if it is done right," in that under certain conditions the club achieves greater total welfare in the long run in the presence of internal politics than when internal politics is absent, if unanimity voting rule is used to admit new members. Moreover, "too much politics is surely a bad thing," in that the club obtains low welfare in the long run when politics becomes very intense. In that case, majority voting rule is better than unanimity voting rule because the club can suffer more welfare loss from politics under unanimity voting.

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1 Introduction

The long run health and survival of an organization depends crucially on its ability to consistently attract and keep high caliber new members, because existing members inevitably have to exit the organization for retirement or other reasons. Except for rare cases, internal politics is an important part of life in organizations, whereby different groups of people vie for control over the decision making power of the organization. Therefore, existing members will look at not only a candidate's qualification when deciding whether to admit him into the organization, but also at how his joining the organization affects the organization's future power structure. In this paper, we develop a stylized model to study how internal politics affects an organization's hiring of new members and investigate the implications of the dynamic interactions between internal politics and hiring of new members on the organization's long run outcomes and welfare.

We consider an organization with a fixed size in which one of the incumbent members retires in each period and the incumbent members vote to admit a candidate to fill the vacancy. Every player has two characteristics: *quality* and *type* in the internal politics of the organization. A player's *quality* represents his skills, prestige or resources that are valuable to every member of the organization. However, internal politics often is anchored on things other than quality, such as race, gender, ideology, specialization (e.g., theorists versus empiricists), or personality. While the political structure of many organizations is often quite complicated, for simplicity we suppose that *every player (incumbent member or candidate) belongs to one of two types: left or right.* The majority type in each period controls the decision-making power of the organization in that period. In particular, we consider *distributive politics* in the sense that there is a fixed amount of rent (e.g., research funds, perks, prestigious positions) in each period that can be distributed to the members of the organization. Thus, the majority type in each period controls the rent allocation and distributes it among members of its type.

Real world organizations that fit our stylized model include academic departments, social clubs, professional societies, condominium associations and partnership firms, etc. We believe the insight of the model also applies to organizations that fit some but not all features of the model. For example, in boards of directors of many public firms, non-profit organizations and local governments (e.g., education board in a city), even though incumbent members do not directly select new members, quite often they do have substantial influence in the selection process. Then internal politics would still affect hiring of new members.

For the ease of exposition, we call the organization a "club". In our model, the club's welfare is independent of its political structure (type profile) since the amount of rent in each period is fixed. Its per period welfare is simply the average quality of the club members, and depends on the admission policies under which the club members are selected. In the first best solution, the social planner of the club optimally trades off the benefits from setting a high standard (same for both types of candidates) to get more qualified candidates and the costs of delay. In another benchmark, suppose there is no internal politics in the club (when there is no rent to grab). In this case, all incumbent members have the identical preference and will choose the same admission standard for both types of candidates. We call the equilibrium in this case the "*harmonious equilibrium.*' However, in the harmonious equilibrium, the optimal admission policy is inefficient. The reason is that there is an "intertemporal free riding" problem in the sense that the incumbent members in the current period do not take into account the effects of their admission decisions on future generations of club members. Consequently all incumbent members search less relative to the efficient level by setting admission standards inefficiently low.

In the presence of internal politics, the incumbent members treat different types of candidates differently, and the club's admission policy in each period depends on its power structure in that period. In choosing their strategies, the incumbent members not only need to calculate the benefits and costs from admitting a candidate of a given quality, but also need to take into account how the type of the admitted candidate affects the power structure of the club in the future. To simplify matters, we focus on the Markov equilibria of the dynamic game in which the incumbent members' strategies only depend on the current period type profile of the club. We characterize the conditions for a Markov equilibrium of the general model and provide a method to solve for the long run stationary outcome and welfare of the club.

We then solve the model of a three member club with uniformly distributed quality, under both majority and unanimity voting rules in selecting new members. Under either voting rule, the solution of the model crucially depends on the value of a variable which is a function of the model's primitive parameters. This variable can be interpreted as the *degree of incongruity* of the club. This variable is smaller, or the club is more congruous, when the rent to fight for in each period is smaller (so the gain of internal politics is smaller), or when the uncertainty over candidate quality is greater (so searching for good candidates is more important relative to rent grabing), or when the delay cost is higher (so the cost of internal politics is larger).

Under majority rule, the majority type in each period can decide on the admission standards for the two types of candidates. In this case the model has two kinds of equilibria, depending on the club's degree of incongruity. When the club is relatively congruous, a "Collegial equilibrium" arises in which even though the majority type discriminates against the opposite type of candidates, both types of candidates have chances to be selected into the club so the club experiences power switches over time. When the club is relatively incongruous, a "glass ceiling equilibrium" arises in which when the majority type has two members (i.e., in a contentious state), the majority type does not admit candidates of any quality from the opposite type. In such an equilibrium, the club will never experience power switches: the type that is initially in power will forever control the club and the minority member will never have any saying in the internal politics of the club. In both equilibria, the standard bias (relative to the harmonious equilibrium) is greater in the contentious states than in the homogenous states (where the club contains just one type), because politics is more intense in the contentious states. Moreover, in both equilibria the long run welfare of the club is lower than that in the harmonious equilibrium, reflecting the cost of internal politics.

Under unanimity voting rule, both types of incumbent members can veto a candidate in the contentious states. Thus coordination and commitment play important roles in determining equilibrium outcomes. As a result, equilibrium characterization is more complicated than under majority voting. As under majority voting, the model under unanimity voting has a Collegial-kind equilibrium (called "reverse Collegial equilibrium" in the sense that the majority type candidate is discriminated against) when the club is relatively congruous, and a glass-ceiling equilibrium when the club is relatively incongruous. Besides these two equilibrium, there can also exist an "exclusive" equilibrium, a "minority tyranny" equilibrium and a "highly political equilibrium" when the club is relatively incongruous. In the exclusive equilibrium, when all the incumbent members of the club are of one type, they never admit candidates of the other type, so the club stays in this state forever. Since the contentious states will eventually migrate to the homogenous states, in the exclusive equilibrium, the club will be in one of the homogenous states in the long run. In the minority tyranny equilibrium, the minority member always vetoes candidates of the majority type in the contentious states, so only candidates of the minority type can be admitted. In such an equilibrium, as the homogenous states will eventually migrate to the contentious states, the club switches back and forth from one contentious state to another forever. In the highly political equilibrium, both types of candidates can be admitted with positive probabilities as in the Collegial equilibrium, hence the club can be in any of the states in the long run. However, unlike in the Collegial equilibrium, the politics in the contentious states become so bad that incumbent members of both types veto candidates of the opposite type with a very high probability, so the club can be in the impasse for a long time.

Which voting rule is better? Comparing the long-run equilibrium outcomes under majority and unanimity voting rules, we find that when the club is relatively congruous, unanimity voting rule is better than majority voting rule. Under both rules, the Collegial-kind equilibrium is the unique equilibrium. Under unanimity voting rule, the equilibrium standard tends to be higher than that under majority rule, because both type incumbents have to agree on the admission of a candidate. This offsets to some extent the intertemporal free riding problem facing the incumbent members, thus increases the club's long run welfare. In fact, because of this effect, the Collegial equilibrium under unanimity voting can yield greater long run welfare than the harmonious equilibrium. Therefore, "some politics can be a good thing if it is done right." On the other hand, when the amount of rent is large, or when search is not very costly, or when the quality variance is small, then unanimity voting rule is worse (at least weakly) than majority voting rule. All the extreme equilibria under unanimity voting yield lower long run welfare than the glass ceiling equilibrium under majority voting, and all of these equilibrium outcomes are worse than that in the harmonious equilibrium. Thus, "too much politics is surely a bad thing."

In the existing literature, Athey, Avery and Zemsky (2000) and Sobel (2000, 2001) also study how admission standards of new members evolve over time in organization. In Athey *et al* (2000), a single firm chooses a dynamically optimal promotion policy to promote low level employees to fill the vacancies of upper level management positions. Employees differ in their abilities (as qualities in our model) and belong to one of the two types (also as in our model). The surplus the firm gets from promoting a low level employee to an upper level position depends on his ability and the mentoring he gets from existing upper level managers of his same type. Thus, the firm trades off the benefit of better mentoring by promoting low level employees of the majority type in the upper level managers and the cost of lower abilities by digging too deep in the pool of low level employees in that type. In Sobel (2000, 2001), candidates wanting to join an organization and the incumbent members ("elite") of the organization have muli-dimensional quality characteristics, and outside judges who have different preferences over quality characteristics rank a candidate relative to the elite members of the organization. A candidate is admitted according to an exogenous admission standard, whereby at least n judges must rank him at least as highly as the r-th member of the current elite. Sobel (2000, 2001) identifies conditions under which standards fall or rise over time. Our model differs from these existing works in several aspects. First, we focus on the dynamic effects of internal politics on admissions of new members. Secondly, one of our objectives is to construct a model that is suitable for welfare analysis, so that organizational design questions such as what is the optimal admission policy can be addressed. For their different purposes, the above mentioned models by construction do not allow welfare analysis. Thirdly, while Athey *et al* (2000) and Sobel (2000, 2001) all have a fixed body of decision-makers on admission of new members, in our model the incumbent members who make admission decisions in each period change over time.

Our paper is also related to two recent papers by Barbera, Maschler, and Shalev (2001) (henceforth BMS) and Granot, Maschler, and Shalev (2002) (henceforth GMS), both of which study a club's dynamic process of admitting new members through strategic voting by its current members. Unlike our paper, each club member has an individual preference over candidates (they are either his friend or enemy), but there is no quality or ability of a candidate that all members value. Another difference with our model is that these two papers do not consider replacement of incumbent members and the candidate population is fixed and finite. They are interested in showing that under different admission rules (quota-1 rule in BMS and unanimity rule in GMS) various equilibrium outcomes can arise due to strategic voting by incumbent members. Other related literature includes Roberts (2001), who analyzes the effect of majority voting on the dynamics of group size; and Ellickson, Grodal, Scotchmer, and Zame (1999, 2001), who study endogenous club formation in a general equilibrium framework in which private consumption goods and club memberships are priced and traded.

The rest of the paper is organized as follows. The next section presents the model, and Section 3 solves for the first best solution of the club as a benchmark case, and Section 4 solves for the "harmonious equilibrium" in a politics-free world as another benchmark case. We then characterize the conditions for a Markov equilibrium of the general model in Section 5 and provide a method to solve for the long run stationary outcome and welfare in Section 6. In Sections 7 and 8, we then solve for the symmetric equilibria of a specialized model of a three member club with uniformly distributed quality. Section 7 considers the case of majority voting to admit new members, and Section 8 studies the case of unanimity voting rule. Section 9 contains concluding remarks.

2 The Model

We consider a model with discrete time and infinite horizon. At the beginning of the game, a club has $2n+1$ incumbent members. Each period is divided into three stages. In the first stage, the $2n+1$ incumbent members must select one new member from a large pool of outside candidates who want to join the club. After the admission of a new member, in the second stage one of the incumbent members is chosen randomly to exit the club permanently for exogenous reasons (e.g., natural death, family reasons, retirement). For a member who exits the club, his payoff outside the club is normalized to zero. In the third stage, the $2n$ remaining incumbent members plus the new member together decide on club politics. The details of the candidate selection stage and the club politics stage will be specified later. The same process repeats each period infinitely. For simplicity, we assume there is no discounting.[1] We also assume that the demand for the club membership is sufficiently strong that it is always desirable to join the club.

The sequence of move within each period as specified above is convenient for our analysis, because it ensures that there are an odd number of voting members in both the candidate selection stage and the club's political decision stage. One can think of other alternative sequences of move, but our results are quite robust in this aspect. For example, suppose at the beginning of each period one of the $2n+1$ incumbent members randomly exits the club (e.g., one faculty member retires in June), and a new member who was admitted in the previous period actually joins the club (e.g., a new faculty member arrives in September). The $2n+1$ members then decide on club politics and admission of a new member for the following period (e.g., recruiting season is in January). As will be clear, our analysis will be completely unchanged with this sequence of move. In other alternative sequences of move that result in an even number of voters in a certain stage (e.g., admitting a new member for the current period and then deciding on club politics),

[1] In our model random exiting the club serves as the role of discounting, thus no discounting over time is needed.

we need specify some tie-breaking rules but our qualitative results should still hold.

A player, either an incumbent member or a candidate, is characterized by his quality and his type. A player's quality, denoted by v, represents his skills, prestige or resources that he can bring to the club and are valuable to the whole club. In the public good spirit of club, we suppose that a player of quality v brings a common value of v per period to *every member* in the club including himself, so his total contribution to the club value per period is $(2n+1)v$. Players in the population differ in quality. For the population, suppose v is distributed according to a distribution function $F(v)$ on $[\underline{v}, \bar{v}]$, where $0 \leq \underline{v} < \bar{v}$, with an everywhere positive density function $f(v) > 0$. When in a period the club's members have qualities v_k, $k \in 1, 2, ..., 2n+1$, we define the club's per capita value in that period as $V = \sum_{k=1}^{k=2n+1} v_k$.

Aside from quality heterogeneity, players belong to one of two types, "left" type and "right" type, that are equally represented in the population. Depending on the applications, type can be interpreted as race, gender, ideology (or party affiliation), or specialization. Type is important because club politics is centered on such characteristics. We consider the situation of distributive club politics in the following sense. In each period, there is a fixed amount of total rent B in the club to be distributed to its members. Depending on the context, rent can take many different forms, such as monetary or non-monetary resources (e.g., research funds, office spaces, other perks), or power and prestige (e.g, chances to become club officials or to represent the club in the public). Distribution of rent B is determined by majority voting of the $2n+1$ members in each period. We focus on majority voting on rent allocation as the club political process because it is perhaps the most common way in collective decision making. Our framework can be used to study other kinds of club political processes such as supermajority voting or non-voting (e.g., bargaining) political process.[2] For simplicity, we suppose that members of the majority type share the rent equally among themselves.

In the above formulation of club politics, two assumptions are important. One is of the nature of "incomplete contracts", namely, there are certain rents of the club that cannot be specified in contracts clear enough among club members and hence are subject to ex post negotiations/politicking by the members. If all resources and rents of the club are completely pre-determined in contracts, then internal politics does not arise at all and all candidates are evaluated based on their qualifications. This is the harmonious equilibrium to be studied in Section 4.[3] Another important assumption is that by distributive politics, the rent available to the controlling type is private good so that each member of the majority type gets a smaller share of

[2] Equivalently, one can imagine that the club elects a chairman or president by majority voting, who then decides on rent distribution. The elected official is loyal to his "party", and distributes the rent to members of his type only.

[3] As our results will show, it is not always in the best interest of the club to pre-determine the rent allocation (say, equal distribution among all members) even if all rents are contractible.

the total rent as the majority increases. This implies that majority type members would favor candidates of their opposite type if they are assured of keeping control over the internal politics of the club. Alternatively, one can imagine the possibility that in addition to his quality as a public good to the whole club, a candidate brings a common value (a public good) to every member of *his own type*, thus majority type members would favor candidates of their own type. This and other possibilities have to be left for future research (see the Conclusion for more discussions of possible extensions).

We now specify the selection stage of new members. In each round of the selection stage, a candidate is randomly drawn from the population. His quality and type are then revealed to the incumbent members, who then vote whether to accept him as a new member. The voting rule is such that the candidate is admitted if and only if at least m incumbent members vote yes, where $m \geq n + 1$. If a candidate gets m or more yes votes, then he is admitted to the club and the selection stage of the current period is over. If a candidate does not get the required m yes votes, then the club draws another candidate from the population and uses the same selection procedure to decide whether to admit him. This selection process continues until a candidate is admitted. We suppose that each selection round imposes a cost of $\tau > 0$ to every incumbent member.[4] Such a cost can take many forms, e.g., reviewing files, interviewing, meetings, and opportunity costs of leaving the position vacant. Since selecting a member takes at least one round, we count selection costs only if it takes more than one round.

We suppose that the club's voting rule in admitting new members is fixed at the beginning of the game and cannot be modified later. This is of course for analytical simplicity, but it is also consistent with the observation that many organizations have very strict requirements for changing their chatter rules or constitutions. One question we are interested in is what voting rule in admitting new members is the best for the long run welfare of the club. One imagines that the founders of the club (or the supervisors of the club as in the case of academic departments) would want to ensure that the club commits to the optimal rule.[5]

All players in the model are assumed to be risk neutral. They maximize their expected utility when choosing their strategies in the candidate selection game of the club.

Our formulation of club politics greatly facilitates welfare comparison, because the value of the club does not directly depend on its type composition. The per capita net value of the club in a given period can be simply defined as the per capita value minus the per capita search cost. Since both the value of quality and the search cost are common to all members and the club size

[4]What is important here is that every member with some decision making power must incur the search cost. It is immaterial that those who do not have any decision making power, such as the minority incumbents under majority voting, do not participate in selecting new members and do not pay the search cost.

[5]For example, Barzel and Sass (1990) provide evidence that developers of condominiums choose voting rules for condominium homeowner's associations to maximize the value of condominium to potential homeowners.

is fixed, the total value (or, the total net value) of the club are simply $(2n+1)$ times the per capita value (or, the per capita net value). Thus we focus on the per capita value and net value as measures of club welfare or efficiency.

Since both qualities and types of candidates are uncertain before they arrive, the club's value and type composition are stochastic over time. Our analysis will focus on the long run (stationary) behavior of these stochastic processes.

3 The First Best Solution

In this section, we solve for the first best solution for the club as a benchmark case for welfare comparison. Since the total amount rent available in the club in each period is fixed, it will be ignored in the welfare calculation throughout the paper.

Since the two types are symmetric, the social planner of the club should have the same admission policy for both types. It is easy to see that the social planner's optimal admission policy should take the following form: admit a candidate if and only if his quality is at least v^*. Since every member of the club is admitted by such a policy, the club's *expected value per period per capita* is $(2n+1)E[v|v \geq v^*]$.[6] To calculate the expected search cost in each period, note that the probability that a candidate is admitted is $x^* = 1 - F(v^*)$. Hence the expected delay in each period is

$$E[d^*] = \sum_{d=1}^{\infty} x^*(1-x^*)^d d = (1-x^*)/x^* = F(v^*)/(1-F(v^*))$$

The club's *expected net value per period per capita* is therefore $(2n+1)E[v|v \geq v^*] - \tau F(v^*)/(1-F(v^*))$. Maximizing this function, we have (proof omitted)

Proposition 1 *In the first best solution,*

(i) When $\tau \geq (2n+1)(Ev - \underline{v})$, the club admits any candidate (i.e., $v^* = \underline{v}$).

(ii) When $0 < \tau < (2n+1)(Ev - \underline{v})$, the club admits candidates whose quality is above v^*, where v^* is the unique solution to

[6] Alternatively, one can calculate the expected per capita value of hiring a new member with quality v to the social planner as follows:

$$\left[1 + \frac{2n}{2n+1} + \left(\frac{2n}{2n+1}\right)^2 + ...\right] v = (2n+1)v$$

This is because a new member of quality v contributes a per capita value of v in each period he remains in the club, and he is in the club for sure in the period he is admitted and has a survival chance of $2n/(2n+1)$ in each of the future periods.

$$\tau = (2n+1)\left[\int_{v^*}^{\bar{v}} v dF(v) - v^*\left(1 - F(v^*)\right)\right] \tag{1}$$

The solution v^* is strictly decreasing in τ.

This is intuitive. In the optimal policy, the social planner trades off the benefit of setting a high admission standard to get more qualified candidates and the cost of delay. Equation (1) says that on the margin, she must be indifferent from accepting a candidate with quality v^* to avoid additional search cost and rejecting him to search for more qualified candidates. When the unit search cost τ is not too large, then the social planner has an optimal interior searching rule: it will search until a candidate's quality is above a pre-fixed level v^*. When search is very costly, $\tau \geq (2n+1)(Ev - \underline{v})$, the club admits any candidate to avoid paying the search cost.[7]

For concreteness and for later comparisons, we consider the following case. Let $n = 1$, so the club has three members. Suppose v is uniform on $[\underline{v}, \bar{v}]$. Let $a \equiv \bar{v} - \underline{v}$ be the spread of the quality distribution. For welfare analysis and comparative statics, we keep $Ev = (\underline{v} + \bar{v})/2$ fixed, because Ev enters payoff functions linearly due to risk neutrality and thus has no effects on equilibrium outcomes and trivial effects on welfare.

By Proposition 1, when $\tau \geq 3(Ev - \underline{v}) = 3(\bar{v} - \underline{v})/2 = 3a/2$, $\hat{v} = \underline{v}$, so the club's expected net value in the first best solution is $3(\bar{v} + \underline{v})/2$. When $0 < \tau < 3(\bar{v} - \underline{v})/2 = 3a/2$, Equation (1) can be reduced to $(\bar{v} - v^*)^2 = 2(\bar{v} - \underline{v})\tau/3$. Hence we have $v^* = \bar{v} - \sqrt{2(\bar{v} - \underline{v})\tau/3} = \bar{v} - \sqrt{2a\tau/3}$. Then the probability that a candidate is admitted in the first best solution can be expressed as $x^* = (\bar{v} - v^*)/a = \sqrt{2\tau/(3a)}$. This has a very simple interpretation. The smaller a is, the smaller is the benefit of searching for one more round.[8] Thus, the admission probability will be higher (or the admission standard will be lower) if the unit search cost τ is higher or the quality distribution has a smaller spread. The club's expected net value in the first best solution can be calculated as $U^* = 3Ev + 1.5a - \sqrt{6a\tau} + \tau$. It can be easily verified that the expected net value is increasing in a and decreasing in τ.

4 The Harmonious Equilibrium

When $B = 0$ or equivalently when the club's rent is pre-determined and not subject to the internal politicking of its members, internal politics has no importance to club members. In such a case all incumbent members have the identical preference over admission policies. We assume that they all vote sincerely according to their preferences (see Section 5 for justifications). Then they

[7] In the other extreme (and trivial) case when $\tau = 0$, the club only admits candidates with the highest quality, i.e., $v^* = \bar{v}$.

[8] This is analogous to option value being increasing in the variance of the return of the underlying asset.

only need to solve for the optimal admission policy that maximizes their payoffs. Consequently the voting rule to admit new members is irrelevant. We call the equilibrium in this case the "harmonious equilibrium." This can serve as a useful benchmark, because it represents a politics-free world. Comparing it with equilibrium outcomes when internal politics is present reveals the effects of internal politics.

The incumbent members in the harmonious equilibrium need to solve an optimal stopping problem, hence the optimal admission policy should take the following form: admit a candidate if and only if his quality is at least $\hat{v}$. Since internal politics is irrelevant, this admission standard should be independent of the type profile of the club and the candidate types. Moreover, since the incumbents' qualities are fixed in any given period at the time they are selecting new members, the quality profile does not have any effect on the incumbents' optimal admission policy. Therefore, the incumbent members' optimal admission policy is constant over time.

Note that the expected value to an incumbent member if a candidate with quality $\hat{v}$ is admitted can be calculated as follows:

$$\frac{2n}{2n+1}\left(1+\frac{2n-1}{2n+1}+\left(\frac{2n-1}{2n+1}\right)^2+....\right)\hat{v}=n\hat{v}$$

where $\frac{2n}{2n+1}$ is the probability that the incumbent member remains in the club in the current period after admitting the candidate. Conditional on that he does, he gets a value of $\hat{v}$ in the current period from admitting the candidate. In the next period, he gets the value $\hat{v}$ only if he and the candidate-turned new member both remain in the club, which occurs with probability $\frac{2n-1}{2n+1}$, so on and so forth. The expected sum of this value stream turns out to be $n\hat{v}$.

Let w be the *expected net value* an incumbent member can get from selecting a new member using the optimal rule. Clearly, $w \in [\underline{v}, \bar{v}]$.[9] By the definition of $\hat{v}$, it must be that

$$n\hat{v} = \max\{w-\tau, n\underline{v}\} \tag{2}$$

When $w-\tau \geq n\underline{v}$, this says that if the candidate's quality happens to be $\hat{v}$, the incumbent members must be indifferent between admitting him now (i.e., receiving value $n\hat{v}$) and rejecting and waiting to see another candidate. In the latter case, an incumbent will receive a value of w (from the same optimal admission policy next round) but will incur the waiting cost of τ. When $w-\tau < n\underline{v}$, waiting never makes sense so the club should admit any candidate, that is, set $\hat{v}=\underline{v}$.

By the definition of w, we have

$$w = n\int_{\hat{v}}^{\bar{v}} v dF(v) + F(\hat{v})(w-\tau) \tag{3}$$

[9]The reason $w \geq \underline{v}$ is that the club can always admit everybody (i.e., $\hat{v}=\underline{v}$), in which case $d=0$.

where the first term is the expected value in the event that the candidate's quality is above $\hat{v}$ (so he is admitted), and the second term is the expected net value in the event that the candidate's quality is below $\hat{v}$ (so the club has to search further).

Equations (2) and (3) define the optimal $\hat{v}$ and the resulting expected net value w. We have the following result.

Proposition 2 *The club's optimal admission policy in the harmonious equilibrium can be characterized as follows:*

(i) When $\tau \geq n(Ev - \underline{v})$, the club admits any candidate (i.e., $\hat{v} = \underline{v}$).

(ii) When $0 < \tau < n(Ev - \underline{v})$, the club admits candidates whose quality is above $\hat{v}$, where $\hat{v}$ is the unique solution to

$$\tau = n \left[\int_{\hat{v}}^{\bar{v}} v dF(v) - \hat{v} \left(1 - F(\hat{v})\right) \right] \tag{4}$$

The solution $\hat{v}$ is strictly decreasing in τ.

(iii) When $\tau < (2n+1)(Ev - \underline{v})$, the admission standard in the harmonious equilibrium is strictly lower than the first best level.

Proof: See the Appendix.

The characterization of the harmonious equilibrium in Proposition 2 is easy to understand. What is interesting is that even in a politics-free world, the club's admission policy is inefficient. Comparing Equations (1) and (4) reveals the source of inefficiency. In the harmonious equilibrium, an incumbent member only gets a marginal benefit of $n\beta$ from setting an admission standard $\hat{v}$, where β is the expression in the RHS of Equations (1) and (4), while the social planner's marginal benefit from setting an admission standard v^* is $(2n+1)\beta$. Facing the same marginal search cost, an incumbent member in the harmonious equilibrium thus sets a lower standard than the social planner. This is similar to the under-provision of public goods in the standard static model of clubs. However, in our model inefficiency does not come from free-riding among incumbent members in a given period. The joint surplus of all incumbent members in any given period is maximized in the harmonious equilibrium.[10] The source of inefficiency in the harmonious equilibrium is "*intertemporal free riding*", because incumbent members in the current period do not take into account the benefits of having high quality new members to the

[10] In Equations (2) and (3), multiplying the value terms and the cost term by $2n+1$ leads to the same equation (4) and gives the same solution.

future generations of club members. Thus, they search less relative to the efficient level by having lower admission standards.

When the club adopts the optimal admission policy every period, every member of the club is admitted by such a policy in the long run.[11] In this case, the expected quality of a new member is $E[v|v \geq \hat{v}]$, so the club's *expected value per period per capita* is simply $(2n+1)E[v|v \geq \hat{v}]$. To calculate the expected search cost, note that the probability that a candidate is admitted is $\hat{x} = 1 - F(\hat{v})$. Hence the expected delay is

$$E[\hat{d}] = \sum_{d=1}^{\infty} \hat{x}(1-\hat{x})^d d = (1-\hat{x})/\hat{x} = F(\hat{v})/(1-F(\hat{v}))$$

Therefore, the club's *expected net value per period per capita* is $(2n+1)E[v|v \geq \hat{v}] - \tau F(\hat{v})/(1 - F(\hat{v}))$. By Proposition 2, since $\hat{v} < v^*$ and by definition v^* maximizes the expected net value, the expected net value in the harmonious equilibrium is lower than that in the first best solution.

Consider the case with $n = 1$ and v is uniformly distributed on $[\underline{v}, \bar{v}]$. By Proposition 2, when $\tau \geq Ev - \underline{v} = (\bar{v} - \underline{v})/2 = a/2$, $\hat{v} = \underline{v}$, so the club's expected net value in the harmonious equilibrium is $3(\bar{v} + \underline{v})/2$. The interesting case is when $0 < \tau < (\bar{v} - \underline{v})/2 = a/2$. Equation (4) becomes $(\bar{v} - \hat{v})^2 = 2a\tau$, so $\hat{v} = \bar{v} - \sqrt{2a\tau}$, and the probability that a candidate is admitted in the harmonious equilibrium is $\hat{x} = (\bar{v} - \hat{v})/a = \sqrt{2\tau/a}$. Compared with the first best solution, we can see that $\hat{v} < v^*$ and $\hat{x} > x^*$: the admission standard is lower in the harmonious equilibrium than in the first best. The club's expected net value in the harmonious equilibrium is $U^h = 3Ev + 1.5a - 2\sqrt{2a\tau} + \tau$. The comparative statics in the harmonious equilibrium are identical to those in the first best: as a increases or and τ decreases, the admission standard becomes higher (admission probability x is smaller) and the expected net value increases.

5 Equilibrium Conditions

In this section we suppose $B > 0$ and characterize the conditions for an equilibrium of the candidate selection game. We first introduce some notation. Let the type profile of the club be represented by a single variable $I \in \{0, 1, 2, ..., 2n+1\}$ indicating the number of right types among the club's incumbent members. We will call I the "state" of the club. Let $b \in \{L, R\}$ denote the type of an incumbent member, and $b' \in \{l, r\}$ denote the type of a candidate. Let $\sigma = (v_j^l, v_j^r)(t)$ be a club's admission policy over time $t = 1, 2, ...$, where $v_j^l(t)$ (resp., $v_j^r(t)$) is the minimum quality for a left (resp., right) type candidate to be admitted when the club is in state $j \in \{0, 1, 2, ..., 2n+1\}$ at time t. At a time t_0, suppose the quality profile of the incumbent

[11]How the original members at the birth of the club are selected is not important, because with probability one they all exit the club in finite time.

members is $\{v_k\}_{k=1}^{k=2n+1}$ and the state is i. Note that the state of the club at $t > t_0$, I_t, only depends on the initial state i at t_0 and the admission policy σ.

For a given admission policy σ, let us calculate the expected payoff of an incumbent member, say $k = 1$, who is of type $b \in \{L, R\}$. Denote this expected payoff as $Eu_1(t_0, \{v_k\}, b, i, \sigma)$. For $t = t_0 + 1$, member 1's expected payoff is

$$Eu_1(t = t_0 + 1) = \frac{2n}{2n+1}\left[v_1 + \frac{2n-1}{2n}\sum_{k=2}^{2n+1} v_k + E[v_{\text{new}}^t|i,\sigma] + E[\mu^t|i,\sigma] - E[d^t|i,\sigma]\right]$$

The interpretation is as follows. The term $\frac{2n}{2n+1}$ is the probability that member 1 survives one period, otherwise member 1 exits the club and gets zero payoff. In the square bracket, v_1 is member 1's own quality, $\frac{2n-1}{2n}$ is the probability that any other member $k > 1$ survives one period conditional on member 1 survives so the second term is the expected total value to member 1 from surviving incumbent members. The term $E[v_{\text{new}}^t|i,\sigma]$ is the expected quality of the newly admitted member in the period, which only depend on the current state of the club and the admission policy. Depending on the types of the exiting member and the newly admitted member, the state of the club may change. The next term $E[\mu^t|i,\sigma]$ represents member 1's expected rent in this period, which only depend on the current state of the club and the admission policy. Finally, $E[d^t|i,\sigma]$ is the expected search cost to member 1 in this period, also only depends on the current state of the club and the admission policy.

Similarly, member 1's expected payoff in the next period $t = t_0 + 2$, $Eu_1(t = t_0 + 2)$, is given by

$$\left(\frac{2n}{2n+1}\right)^2\left[v_1 + \frac{2n-1}{2n}\left[\frac{2n-1}{2n}\sum_{k=2}^{2n+1} v_k + E[v_{\text{new}}^{t_0+1}|i,\sigma]\right] + E[v_{\text{new}}^t|i,\sigma] + E[\mu^t|i,\sigma] - E[d^t|i,\sigma]\right]$$

Here every term is similarly defined as in $Eu_1(t = t_0 + 1)$. It is worth pointing out that $\frac{2n-1}{2n}\sum_{k=2}^{2n+1} v_k + E[v_{\text{new}}^{t_0+1}|i,\sigma]$ is the expected total value to member 1 from other incumbent members at the beginning of the period $t = t_0 + 2$. Multiplied by the survival probability $\frac{2n-1}{2n}$ conditional on member 1 remaining in the club, the second term in the square bracket is the expected total value to member 1 from surviving incumbent members in period $t = t_0 + 2$.

By deduction, the expected payoff of member 1 who is type b can be written as

$$\begin{aligned}
Eu_1(t_0, \{v_k\}, b, i, \sigma) &= \sum_{t=t_0+1}^{\infty} Eu_1(t) \\
&= \left(\frac{2n}{2n+1} + \left(\frac{2n}{2n+1}\right)^2 + ...\right)v_1 + \left(\frac{2n-1}{2n+1} + \left(\frac{2n-1}{2n+1}\right)^2 + ...\right)\sum_{k=2}^{2n+1} v_k + \pi_i^b(\sigma)
\end{aligned}$$

$$= \quad 2nv_1 + \frac{2n-1}{2}\sum_{k=2}^{2n+1} v_k + \pi_i^b(\sigma) \tag{5}$$

where $\pi_i^b(\sigma)$ contains all the terms about the expected qualities of newly admitted members in each period, the expected rent member 1 gets in each period, and the expected search cost in each period. In other words, $\pi_i^b(\sigma)$ is the expected payoff a type b incumbent member in state i can get through the club's admissions of new members in the current and future periods. As a default rule, we set $\pi_i^b = 0$ if (i) $i = 0$ and $b = R$; or (ii) $i = 2n+1$ and $b = L$.

A key observation is that $\pi_i^b(\sigma)$ is independent of the quality profile of the club $\{v_k\}$ at $t = t_0$, and the quality profile enters each incumbent member's expected payoff as a constant. Thus, the incumbent members choose strategies to maximize $\pi_i^b(\sigma)$ since the quality profile is fixed. To make the model tractable, we focus on equilibria in which players use Markov strategies that only depend on the state variable—the type profiles of the club. Without putting restrictions on strategies, the model becomes trivial in the following sense. In any given period, if every incumbent member votes "no" on *any* candidate, then it is indeed an equilibrium that no candidate will be admitted forever. But in this equilibrium every incumbent member gets a payoff of negative infinity (as long as τ is positive)! Using this equilibrium as a punishment, then any outcome can be supported in equilibrium. It does not seem reasonable that players can credibly commit to such punishments. By focusing on Markov strategies that only depend on the type profiles of the club, we rule out history dependent award and punishment schemes. This seems reasonable, especially given that the voting electorate is changing over time in our model. Even with this restriction, the equilibrium strategies and the long run behavior of the club in the model still exhibit interesting dynamics. In this perspective, our results are robust because the equilibrium concept used is the strongest possible in the model.

We suppose that incumbent members do not use weakly dominated strategies in each round of voting.[12] It is needed because in any given period, any voting outcome can be supported in equilibrium if the equilibrium with negative infinity payoff for every player is used as a threat. By this assumption, every incumbent member is going to vote sincerely his true preference. By Equation (5), since $\pi_i^b(\sigma)$ is common to all incumbent members of a same type, incumbent members of a same type have the same preference over the club's admission policy. It follows that incumbent members of a same type always vote as a block and they vote consistently in any given period.

[12]This is a common assumption in the literature to rule out equilibria of coordination failure in voting, i.e., voting "no" on a preferred outcome is a weakly dominated best response if everyone else does so. A "trembling hand" argument ensures that voters do not use weakly dominated strategies because there is always a positive probability that he is pivotal. Alternatively, if incumbent members vote sequentially in each selection round, then they will vote their true preferences as well.

We focus on symmetric equilibria of the game. Since the model is symmetric with respect to the two types, right type members in state i are in the same strategic position as left type members in state $2n+1-i$. We suppose that right type members in state i choose the same strategies as left type members in state $2n+1-i$ in equilibrium. For symmetric equilibria, we only need to specify strategies of the club's incumbent members in states $n+1 \leq i \leq 2n+1$.

Depending on the club's voting rule m and the state I, there are three possible regimes. One regime is called the *"right"* regime, in which $I \geq m$ so the block of right types can decide the admission policy on their own. Another regime is called the *"left"* regime, in which $I \leq 2n+1-m$ so the block of left types can decide the admission policy on their own. A third regime is called the *"balanced"* regime, in which $2n+1-m < I < m$ so that neither block can unilaterally decide the admission policy. Clearly, the balanced regime becomes larger while the other two regimes shrink as m increases. Under majority voting ($m = n+1$), the balanced regime disappears. Under unanimity voting ($m = 2n+1$), all states belong to the balanced regime except for $I = 0$ or $I = 2n+1$.

We call a state $i \geq n+1$ a "right" state, in which the right type incumbents dominate the internal politics and get all the rents of the club. If a club is in the right regime, that it is in a right state. When $m > n+1$, there are right states in which the right type incumbents only control the rent distribution but cannot unilaterally decide on admission of new members.

Suppose the club's admission policy is $\sigma = (v_i^l, v_i^r)$, where v_i^l (resp., v_i^r) is the admission criterion for a left (resp., right) type candidate in each state i. Consider a right state $i \geq n+1$. For a right type incumbent member "A" with quality v_k, let $V_{-k} = \sum_{j \neq k} v_j$, $V_{-k}^R = \sum_{j \neq k, b_j = R} v_j$, and $V_{-k}^L = \sum_{j \neq k, b_j = L} v_j$. We rewrite the A's expected payoff defined in the way of Equation (5) as $Eu_i^R(v_k, V_{-k})$, where the subscript i denotes the current state, the superscript R denotes A's type, and the admission policy σ is suppressed to simplify notation.

In state i, if the club admits a right type candidate with quality v^r in the first selection round, A's expected payoff is

$$\begin{aligned} Eu_i^R(v_k, V_{-k}, v^r, \text{yes}) \quad &= \quad \frac{2n}{2n+1}\left[v_k + \frac{2n-1}{2n}V_{-k} + v^r + \frac{i-1}{2n}\left(\frac{B}{i} + Eu_i^R(v_k, V_{-k} - \frac{1}{i-1}V_{-k}^R + v^r)\right)\right. \\ &+ \quad \left.\frac{2n-i+1}{2n}\left(\frac{B}{i+1} + Eu_{i+1}^R(v_k, V_{-k} - \frac{1}{2n-i+1}V_{-k}^L + v^r)\right)\right] \end{aligned}$$

The interpretation is as follows. Once the right type candidate is admitted, A exits the club with probability $1/(2n+1)$ (in which case he gets a payoff of zero) and remains in the club with a probability of $2n/(2n+1)$. Conditional on A remaining in the club, there are two possible events. One is that the exiting incumbent member is a right type, which occurs with probability $(i-1)/(2n)$. In this event, the state remains at i, so A gets a payoff of $v^r + \frac{B}{i}$ in the current period, and a future payoff of $Eu_i^R(v_k, V_{-k} - \frac{1}{i-1}V_{-k}^R + v^r)$. In another event, the exiting incumbent member is a left type, which occurs with probability $(2n-i+1)/(2n)$. In this event, the state changes to $i+1$, so A gets a payoff of $v^r + \frac{B}{i+1}$ in the current period, and a future payoff of $Eu_{i+1}^R(v_k, V_{-k} - \frac{1}{2n-i+1}V_{-k}^L + v^r)$. Thus, the expression in the square bracket of the RHS of Equation (6) gives A's expected payoff in these two events conditional on him remaining in the club.

Using Equation (5) for $Eu_i^R(v_k, V_{-k} - \frac{1}{i-1}V_{-k}^R + v^r)$ and $Eu_{i+1}^R(v_k, V_{-k} - \frac{1}{2n-i+1}V_{-k}^L + v^r)$, we can simplify the above expression and obtain

$$\begin{aligned} Eu_i^R(v_k, V_{-k}, v^r, \text{yes}) &= 2nv_k + \frac{2n-1}{2}V_{-k} + nv^r \\ &\quad + \frac{i-1}{2n+1}\left(\frac{B}{i} + \pi_i^R\right) + \frac{2n-i+1}{2n+1}\left(\frac{B}{i+1} + \pi_{i+1}^R\right) \end{aligned} \tag{6}$$

Similarly, in a right state $i \geq n+1$, the expected payoff of a left type incumbent member from admitting a right type candidate with quality v^r is given by

$$Eu_i^L(v_k, V_{-k}, v^r, \text{yes}) = 2nv_k + \frac{2n-1}{2}V_{-k} + nv^r + \frac{i-1}{2n+1}\pi_i^L + \frac{2n-i+1}{2n+1}\pi_{i+1}^L \tag{7}$$

In a state $i \geq n+2$, the right type incumbents are safely in control of the political process on rent distribution at least for one period, since a new admission of a left type candidate cannot change the right type's power over rent distribution. In such a state, member A's expected payoff from admitting a left type candidate with quality v^l can be calculated as follows:

$$\begin{aligned} Eu_i^r(v_k, V_{-k}, v^l, \text{yes}) &= 2nv_k + \frac{2n-1}{2}V_{-k} + nv^l \\ &\quad + \frac{i-1}{2n+1}\left(\frac{B}{i-1} + \pi_{i-1}^R\right) + \frac{2n-i+1}{2n+1}\left(\frac{B}{i} + \pi_i^R\right) \end{aligned} \tag{8}$$

That is, conditional on member A remaining in the club, the state will either remain in state i (if the exiting member is a left incumbent) or change to $i-1$ (if the exiting member is a right incumbent other than A). In either case the right type members are still in power, and member A still enjoys a share of rent.

In the state $i = n+1$, if a left type candidate with quality v^l is admitted, a right type incumbent member A's expected payoff is

$$Eu_i^r(v_k, V_{-k}, v^l, \text{yes}) = 2nv_k + \frac{2n-1}{2}V_{-k} + nv^l + \frac{i-1}{2n+1}\pi_{i-1}^R + \frac{2n-i+1}{2n+1}\left(\frac{B}{i} + \pi_i^R\right) \tag{9}$$

The difference with a state $i \geq n+2$ is that conditional on member A remaining in the club, the right type loses control of internal politics in the club and member A gets no rent in the current period if the exiting member is a right incumbent.

Similarly, in a right state i, the expected payoff of a left type incumbent member from admitting a left type candidate with quality v^l is given by

$$Eu_i^L(v_k, V_{-k}, v^l, \text{yes}) = \begin{cases} 2nv_k + \frac{2n-1}{2}V_{-k} + nv^l + \frac{i-1}{2n+1}\pi_{i-1}^L + \frac{2n-i+1}{2n+1}\pi_i^L & \text{, if } i \geq n+2; \\ 2nv_k + \frac{2n-1}{2}V_{-k} + nv^l + \frac{i-1}{2n+1}\left(\frac{B}{i} + \pi_{i-1}^L\right) + \frac{2n-i+1}{2n+1}\pi_i^L & \text{, if } i = n+1. \end{cases} \tag{10}$$

If a candidate is rejected by the club, no matter what the type or quality of the candidate is, a type $b \in \{L, R\}$ incumbent member's expected payoff is simply

$$Eu_i^b(v_k, V_{-k}, \text{no}) = Eu_i^b(v_k, V_{-k}) - \tau \tag{11}$$

This is because after rejecting the candidate, the club continues the selection process in the same manner in the current period, so an incumbent member's expected payoff since the next selection round remains at $Eu_i^b(v_k, V_{-k})$. Since each incumbent incurs a search cost of τ for one round of delay, his expected payoff from turning down a candidate is given by $Eu_i^b(v_k, V_{-k}) - \tau$.

Given the admission policy (v_i^l, v_i^r), we can now calculate the expected payoff of a type $b \in \{R, L\}$ incumbent member in a right state $i \geq n+1$ as follows

$$\begin{aligned} Eu_i^b(v_k, V_{-k}) &= 0.5\left[\int_{v_i^r}^{\bar{v}} Eu_i^b(v_k, V_{-k}, v_i^r, \text{yes})dF(v_i^r) + F(v_i^r)(Eu_i^b(v_k, V_{-k}) - \tau)\right] \\ &\quad +0.5\left[\int_{v_i^l}^{\bar{v}} Eu_i^b(v_k, V_{-k}, v_i^l, \text{yes})dF(v_i^l) + F(v_i^l)(Eu_i^b(v_k, V_{-k}) - \tau)\right] \end{aligned} \tag{12}$$

Thus, an incumbent's expected payoff is the sum of his expected payoffs in the two events depending on the type of the first candidate the club considers. If the first candidate is the right type, a right (resp., left) type incumbent's expected payoff is given by Equation (6) (resp., 7) when the candidate is qualified, and it is $Eu_i^R(v_k, V_{-k}) - \tau$ (resp., $Eu_i^L(v_k, V_{-k}) - \tau$) when the candidate is not qualified. So the expression in the first (resp., second) square bracket of the RHS of Equation (12) is an incumbent's expected payoff conditional on the first candidate is the right (resp., left) type.

Optimal Admission Policies in the Right Regime

In a state $i \geq m$ that belongs to the right regime, the block of right type members need to decide on an admission policy (v_i^l, v_i^r). For an admission policy (v_i^l, v_i^r) to be optimal for a right type incumbent with quality v_k in a state i in the right regime, it must be that, for candidate of types $b' \in \{l, r\}$,

$$v_i^{b'} \begin{cases} = \underline{v} & , \quad \text{if } Eu_i^R(v_k, V_{-k}, v_i^{b'} = \underline{v}, \text{yes}) \geq Eu_i^R(v_k, V_{-k}) - \tau; \\ \in (\underline{v}, \bar{v}) & , \quad \text{if } Eu_i^R(v_k, V_{-k}, v_i^{b'}, \text{yes}) = Eu_i^R(v_k, V_{-k}) - \tau; \\ = \bar{v} & , \quad \text{if } Eu_i^R(v_k, V_{-k}, v_i^{b'} = \bar{v}, \text{yes}) \leq Eu_i^R(v_k, V_{-k}) - \tau. \end{cases} \tag{13}$$

where $Eu_i^R(v_k, V_{-k}, v_i^r, \text{yes})$ is defined by Equation (6) with $v_1^r = v_i^r$; and $Eu_i^R(v_k, V_{-k}, v_i^l, \text{yes})$ is defined by Equation (8) for a state $i \geq n+2$ and by Equation (9) for a state $i = n+1$ with $v^l = v_i^l$.

Optimal Admission Policies in the Balanced Regime

Now we consider the case in which the club is in the balanced regime, $2n+1-m < R < m$. Let $v_i^{b'}$ for $b' \in \{l, r\}$ be the admission policy in a right state $n+1 \leq i < m$ of the balanced regime. For a candidate of type $b' \in \{l, r\}$ with quality $v_i^{b'}$ to be admitted, both the right and left type incumbents have to prefer the admission to going to another selection round. Therefore, an optimal admission policy (v_i^l, v_i^r) must satisfy

$$v_i^{b'} \begin{cases} = \underline{v} & , \quad \text{if } Eu_i^b(v_k, V_{-k}, v_i^{b'} = \underline{v}, \text{yes}) \geq Eu_i^b(v_k, V_{-k}) - \tau \text{ for } b = L \text{ and } b = R; \\ \in (\underline{v}, \bar{v}) & , \quad \begin{array}{l} \text{if } Eu_i^b(v_k, V_{-k}, v_i^{b'}, \text{yes}) \geq Eu_i^b(v_k, V_{-k}) - \tau \text{ for } b = L \text{ and } b = R \\ \text{and with equality for at least one } b; \end{array} \\ = \bar{v} & , \quad \text{if } Eu_i^b(v_k, V_{-k}, v_i^{b'} = \bar{v}, \text{yes}) \leq Eu_i^b(v_k, V_{-k}) - \tau \text{ for } b = L \text{ or } b = R. \end{cases} \tag{14}$$

Conditions for the optimal admission policies in the left regime and in the left states in the balanced regimes can be symmetrically defined. Note that v_k and V_{-k} can be canceled out in both Conditions (13) and (14). Therefore, if the two conditions hold for one right type incumbent member, they hold for every right type incumbent member. Thus, we have the following result:

Proposition 3 *A symmetric equilibrium of the model is an admission policy (v_i^l, v_i^r) with associated value functions (π_i^L, π_i^R) that satisfy the following conditions: (i) $v_i^{b'} = v_{2n+1-i}^{\overline{b'}}$ and $\pi_i^b = \pi_{2n+1-i}^{\bar{b}}$ for all i and b; (ii) Condition (13) for $i \geq m$; (iii) Condition (14) for $n+1 \leq i < m$; and (iv) Equation (12) for $i \geq n+1$.*

6 Long Run Outcome and Welfare

In this section we describe the long run behavior of the club and show how to evaluate its welfare given its admission policy. With an admission policy (v_i^l, v_i^r) in state i, the probability of the newly admitted member being the right type, p_i^r, must satisfy

$$p_i^r = 0.5[1 - F(v_i^r)] + 0.5F(v_i^r)p_i^r + 0.5F(v_i^l)p_i^r$$

That is, the new member can be the right type in one of the three events whose probabilities correspond to the three terms above, respectively: (1) the first candidate is the right type with quality above v_i^r and so is admitted; (2) the first candidate is the right type with quality below v_i^r and so is rejected but the club admits a right type member eventually; and (3) the first candidate is the left type with quality below v_i^l and so is rejected, but the club admits a right type member eventually. Solving for p_i^r, we have

$$p_i^r = \frac{1 - F(v_i^r)}{2 - F(v_i^r) - F(v_i^l)} \tag{15}$$

This is very intuitive. Note that $1 - F(v_i^r)$ is the probability of a right type candidate being admitted and $1 - F(v_i^l)$ is the probability of a left type candidate being admitted in any single round. Since all selection rounds are identical, the chance of the new member being the right type simply depends on the ratio of $1 - F(v_i^r)$ to $1 - F(v_i^l)$. Similarly, the probability of the newly admitted member in state i being the left type p_i^l is given by

$$p_i^l = 1 - p_i^r = \frac{1 - F(v_i^l)}{2 - F(v_i^r) - F(v_i^l)}$$

The evolvement of the state variable, the number of right type members in the club I, constitutes a Markov chain (in fact, a random walk). Its transition probabilities can be calculated as follows: for $i, j \in \{0, 1, 2, ..., 2n+1\}$,

$$p_{ij} = \begin{cases} 0 & , \text{ if } \quad |i-j| \geq 2; \\ p_i^r(1 - \frac{i}{2n+1}) & , \text{ if } \quad j = i+1; \\ p_i^r \frac{i}{2n+1} + p_i^l(1 - \frac{i}{2n+1}) & , \text{ if } \quad j = i; \\ p_i^l \frac{i}{2n+1} & , \text{ if } \quad j = i-1. \end{cases} \tag{16}$$

where p_{ij} is the probability of the state moving from i to j, for $i, j \in \{0, 1, ..., 2n+1\}$. Since only one new member is added to the club and only one incumbent exits the club in one period, the number of right type members can differ by at most one. It increases by one if the new member is the right type which occurs with probability p_i^r, and the exiting member is the left type which occurs with probability $1 - i/(2n+1)$. Similarly, the number of right type members decreases by one if the new member is the left type which occurs with probability p_i^l, and the exiting member is the right type which occurs with probability $i/(2n+1)$. Finally, the number of right type members does not change if the type of the incoming new member is the same as that of the exiting member which occurs with probability $p_i^r \frac{i}{2n+1} + p_i^l(1 - \frac{i}{2n+1})$.

Let $\mathbf{P} = (p_{ij})_{i,j \in \{0,1,...,2n+1\}}$ be the transition probability matrix of the random walk of I. Its stationary probability distribution $\mathbf{Q}$ is given by

$$\mathbf{Q} = \mathbf{P}'\mathbf{Q} \tag{17}$$

where $\mathbf{Q} = \{q_i\}$ and $q_i \in [0, 1]$ is the stationary probability of the state I equal to i for $i \in \{i = 0, 1, ..., 2n + 1\}$ such that $\sum_i q_i = 1$, and $\mathbf{P'}$ is the transpose of $\mathbf{P}$. As we will see, in some cases the random walk process is reducible, in which case Equation (17) can be applied to each recurrent class to derive the stationary probability distribution.

The probability distribution $\mathbf{Q}$ describes the long run behavior of the club's state. Given $\mathbf{Q}$ and the club's admission policy in each state, we can calculate the long run *expected quality* of a representative club member in the club, denoted by s, as follows:

$$s = \sum_{i=0}^{i=2n+1} q_i \left(p_i^r E[v|v \geq v_i^r] + p_i^l E[v|v \geq v_i^l] \right) \tag{18}$$

where for each state i, q_i is the probability of the state I equal to i, p_i^r (resp., p_i^l) is the probability that a newly admitted member being the right (resp., left) type, and $E[v|v \geq v_i^r]$ (resp., $E[v|v \geq v_i^l]$) is the expected quality of a newly admitted right (resp., left) type member. The idea behind the definition of s is that in the long run, every member is admitted in one of the states under the same admission policy. The expression $p_i^r E[v|v \geq v_i^r] + p_i^l E[v|v \geq v_i^l]$ is the expected quality of a new member in a given state i under the admission policy (v_i^l, v_i^r). Taking expectation over the states using the stationary probability distribution thus gives the expected quality of a representative club member in the club.

Let $S = (2n + 1)s$ be the long run "expected value" per capita per period of the club. To calculate the expected search cost in the long run stationary "world" (to avoid confusion about "state" as the state variable), note that for any given state i and admission policy (v_i^l, v_i^r), a candidate is admitted with probability of

$$x_i = 0.5(1 - F(v_i^r)) + 0.5(1 - F(v_i^l)) = 1 - 0.5F(v_i^r) - 0.5F(v_i^l)$$

The expected delay in state i is then given by

$$E[d_i] = \sum_{d=1}^{\infty} x_i(1 - x_i)^d d = (1 - x_i)/x_i$$

Thus, the expected delay in the long run stationary world is

$$D = \sum_{i=0}^{i=2n+1} q_i E[d_i] \tag{19}$$

Now we can define the long run expected net value per capita per period of the club as follows:

$$U = S - \tau D = (2n + 1) \sum_{i=0}^{i=2n+1} q_i \left(p_i^r E[v|v \geq v_i^r] + p_i^l E[v|v \geq v_i^l] \right) - \tau \sum_{i=0}^{i=2n+1} q_i E[d_i] \tag{20}$$

This variable U will be the measure of the club's long run welfare in our analysis below.

7 Majority Voting in a Three Member Club

In this section we consider the following case: (i) $n = 1$, so the club has three members; (ii) $m = 2$ so the club uses majority voting rule to select new members; and (iii) v is uniform on $[\underline{v}, \bar{v}]$ where $0 \leq \underline{v} < \bar{v}$. Due to the majority voting rule in admitting new members, there is no balanced regime and by symmetry we only need to focus on the right regime. Recall that we define $a \equiv \overline{v} - \underline{v}$ as the spread of the quality distribution. To focus our attention, we suppose that the search cost of each selection round τ is less than $a/4$.[13] This assumption appears to be reasonable in most applications, because selection costs involved in recruiting one candidate, such as time costs of reading files and going to meetings, seem to be small relative to the importance of admitting high quality new members.

7.1 Collegial Equilibrium

We analyze possible symmetric equilibria such that $v_i^{b'} \in (\underline{v}, \bar{v})$ for all i and b'. In such an equilibrium, the club migrates from one state to another over time across all states (i.e., irreducible Markov chain) and power switches back and forth between the right types to the left types. We call such an equilibrium "Collegial equilibrium". By Proposition 3, a symmetric equilibrium satisfies Condition (13) with equality for $i = 3, 2$ and $b' = l, r$. After some algebra calculation, we have

$$\frac{2}{3}\left[\frac{3}{2}v_3^r + \frac{B}{3} + \pi_3^R\right] = \pi_3^R - \tau \tag{21}$$

$$\frac{2}{3}\left[\frac{3}{2}v_3^l + \frac{B}{2} + \pi_2^R\right] = \pi_3^R - \tau \tag{22}$$

$$\frac{2}{3}\left[\frac{3}{2}v_2^r + \frac{5B}{12} + \frac{1}{2}\pi_2^R + \frac{1}{2}\pi_3^R\right] = \pi_2^R - \tau \tag{23}$$

$$\frac{2}{3}\left[\frac{3}{2}v_2^l + \frac{B}{4} + \frac{1}{2}\pi_1^R + \frac{1}{2}\pi_2^R\right] = \pi_2^R - \tau \tag{24}$$

By Equation (12), we can obtain, for $i = 3, 2$ and $b = r$,

$$\pi_3^R = \frac{\overline{v} - v_3^r}{3a}\left[\frac{B}{3} + \pi_3^R\right] + \frac{\left[\overline{v}^2 - (v_3^r)^2\right]}{4a} + \frac{v_3^r - \underline{v}}{2a}[\pi_3^R - \tau]$$

[13] If $\tau > a/4$, solutions of the model are more likely to be corner, in the sense that admission of a certain type of candidate is guaranteed regardless of quality. In the extreme, if τ is very large, then the solution becomes the trivial one in that any candidate is admitted. It is worth pointing out that in corner solutions our results still hold qualitatively.

$$+\frac{\overline{v}-v_3^l}{3a}[\pi_2^R+\frac{B}{2}]+\frac{\left[\overline{v}^2-\left(v_3^l\right)^2\right]}{4a}+\frac{v_3^l-\underline{v}}{2a}[\pi_3^R-\tau] \tag{25}$$

$$\begin{aligned}\pi_2^R &= \frac{\overline{v}-v_2^r}{3a}\left[\frac{5B}{12}+\frac{1}{2}\pi_2^R+\frac{1}{2}\pi_3^R\right]+\frac{\left[\overline{v}^2-(v_2^r)^2\right]}{4a}+\frac{v_2^r-\underline{v}}{2a}[\pi_2^R-\tau]\\ &+\frac{\overline{v}-v_2^l}{3a}\left[\frac{B}{4}+\frac{1}{2}\pi_1^R+\frac{1}{2}\pi_2^R\right]+\frac{\left[\overline{v}^2-\left(v_2^l\right)^2\right]}{4a}+\frac{v_2^l-\underline{v}}{2a}[\pi_2^R-\tau]\end{aligned} \tag{26}$$

Also by Equation (12), and using the fact that $\pi_2^L=\pi_1^R$, we have

$$\begin{aligned}\pi_1^R &= \frac{\overline{v}-v_2^r}{3a}\pi_1^R+\frac{\left[\overline{v}^2-(v_2^r)^2\right]}{4a}+\frac{v_2^r-\underline{v}}{2a}[\pi_1^R-\tau]\\ &+\frac{\overline{v}-v_2^l}{3a}[\pi_1^R+\frac{B}{2}]+\frac{\left[\overline{v}^2-\left(v_2^l\right)^2\right]}{4a}+\frac{v_2^l-\underline{v}}{2a}[\pi_1^R-\tau]\end{aligned} \tag{27}$$

Thus, we have a system of 7 equations (21)-(27) with 7 unknowns: $v_2^r, v_2^l, v_3^r, v_3^l, \pi_1^R, \pi_2^R, \pi_3^R$. Define $x_i^{b'} \equiv (\overline{v}-v_i^{b'})/a$ as the probability that a type b' candidate will be admitted in state i. We have the following result.

Proposition 4 *Suppose* $0 < c \equiv B/(12\sqrt{a\tau}) < 0.417$*. Then a Collegial equilibrium exists.*

(i) In the Collegial equilibrium, $x_2^r > x_3^l > \hat{x} = \sqrt{2\tau/a} > x_3^r > x_2^l$*. Thus, standard bias is greater in states* $i=2$ *and* $i=1$ *than in states* $i=3$ *and* $i=0$*.*

(ii) As c *increases, standard bias becomes greater:* $x_2^r/\hat{x}$ *and* $x_3^l/\hat{x}$ *increase in* c *but* $x_3^r/\hat{x}$ *and* $x_2^l/\hat{x}$ *decrease in* c*.*

Proof: See the Appendix.

Proposition 4 says that a collegial equilibrium exists when the variable $c \equiv B/(12\sqrt{a\tau})$ is relatively small. The variable can be interpreted as the club's degree of incongruity. It is small (or, the club is congruous) when the rent or the gain from politicking is not very large (B is relatively small), or when uncertainty of candidate quality is large (a is relatively large), or when delay is costly (τ is relatively large). When a is relatively large, searching for better candidates is important, hence grabing rent through internal politics becomes less important. The last one reflects the effect of unit search cost on internal politics. Internal politics creates biases in admission standard. Thus, a certain type of candidates will face an admission standard higher than the efficient level, resulting in more delays. Therefore, when search is more costly, incumbent members of the majority type will reduce their bias so a collegial equilibrium is more likely to arise. Intuitively, high unit search cost τ increases the cost of internal politics, thus reduces the amount of politicking in equilibrium.

When $B = 0$ (hence $c = 0$), internal politics is irrelevant and the club is harmonious, whereby the admission probability for any type of candidate in any state is $\hat{x} = \sqrt{2\tau/a}$. Compared with this harmonious outcome, admission standards will be biased in equilibria when $B > 0$. Part (i) of Proposition 4 says that *standard bias is greater in states that are more contentious*. When there are two right type members and one left type member (or two left types and one right type) in the club, the majority type members fear that admitting a candidate of the opposite type may twist the balance of power against them and hence will have much higher standard for candidates of the opposite type than for those of their own type. In contrast, when all three members are the right (or left) type, they are safely in control of the power over rent distribution. Since they prefer sharing the rent with fewer members of their own type, they actually will favor candidates of the opposite type and discriminate against those of their own type.

In this model, it is convenient to measure standard bias by the ratio $x_i^{b'}/\hat{x}$, because it is a function of c only. The last statement of Proposition 4 shows that in the collegial equilibrium, standard bias is monotonic in c. By the definition of c, standard bias is greater when power is important (B is large), or when searching for better candidates is not too important (a is small), or when delay in selecting new members is not very costly (τ is relatively small). These comparative statics results are all intuitive.

One may also want to compare the admission standards in the collegial equilibrium with that of the first best solution. Recall that in the first best solution, a candidate of either type is admitted with probability of $x^* = \sqrt{2\tau/(3a)}$, which is lower than $\hat{x} = \sqrt{2\tau/a}$. Thus, immediately from Proposition 4, candidates of the majority type in a contentious state ($i = 2, 1$) and candidates of the minority type in a homogenous state ($i = 3, 0$) face standards lower than the efficient levels. However, candidates of the minority type in a contentious state and those of the majority type in a homogenous state face standards that can be either lower or higher than the efficient level. It depends on c. It is also clear that standard distortions measured by x_2^r/x^* and x_3^l/x^* increase in c.

7.2 Glass Ceiling Equilibrium

We now investigate the possibility that the majority type will never change. In such an equilibrium, if the right types control the rent distribution in the club, then they never yield power to the left types. This takes place when the state is two, the two right type incumbents will not admit an opposite type candidate with any quality, i.e., $v_2^l = \bar{v}$. We call an equilibrium with such properties a "glass ceiling equilibrium", because the type that is in the minority at the birth of the club has no chance of having a saying in the internal politics throughout the life of the club.

In a glass ceiling equilibrium, Equations (21), (22), (23) and (25) should still hold, while Equations (24), (26), and (27) have to be changed. Since $v_2^l = \bar{v}$, by Condition (13), the following condition must hold:

$$\frac{2}{3}\left[\frac{3}{2}\overline{v} + \frac{1}{2}\pi_1^R + \frac{1}{2}\pi_2^R + \frac{B}{4}\right] \leq \pi_2^R - \tau \tag{28}$$

With $v_2^l = \bar{v}$, Equations (26) and (27) are changed to

$$\pi_2^R = \frac{\pi_2^R - \tau}{2} + \frac{\overline{v} - v_2^r}{3a}\left[\frac{5B}{12} + \frac{1}{2}\pi_2^R + \frac{1}{2}\pi_3^R\right] + \frac{\left[\overline{v}^2 - (v_2^r)^2\right]}{4a} + \frac{v_2^r - \underline{v}}{2a}[\pi_2^R - \tau] \tag{29}$$

$$\pi_1^R = \frac{\pi_1^R - \tau}{2} + \frac{\overline{v} - v_2^r}{3a}\pi_1^R + \frac{\left[\overline{v}^2 - (v_2^r)^2\right]}{4a} + \frac{v_2^r - \underline{v}}{2a}[\pi_1^R - \tau] \tag{30}$$

Thus we have 6 equations (21), (22), (23), (25), (29) and (30) with 6 unknowns: $v_2^r, v_3^r, v_3^l, \pi_1^R, \pi_2^R, \pi_3^R$. The solution to this equation system must also satisfy (28) for it to constitute an equilibrium.

Proposition 5 *A glass ceiling equilibrium exists when $c \equiv B/(12\sqrt{a\tau}) > \frac{10}{29} = 0.345$. In this equilibrium, $x_2^r = 2\sqrt{\tau/a} > x_3^l > \hat{x} = \sqrt{2\tau/a} > x_3^r > x_2^l = 0$. Standard bias is increasing in c: $x_3^l/\hat{x}$ increases in c and $x_3^r/\hat{x}$ decreases in c.*

Proof: See the Appendix.

Thus, by Proposition 5, when c is sufficiently large, the club becomes segregated in the sense that the type who is in power at the beginning of the club will remain in power forever. In the glass ceiling equilibrium, the majority right type in the contentious state $i = 2$ will never admit a candidate of the opposite type no matter what his quality is. This implies that if a right type candidate is rejected (because of low quality), on average it takes 2 rounds to get another right type candidate, thus the cost of rejection increases. Consequently, the admission standard for right type candidates must be lowered. Thus, discrimination (difference in standards) between the two types of candidates is the highest in this case. As in the collegial equilibrium, the right type incumbents will favor candidates of the left type more than those of the right type in state $i = 3$. But since they know for sure that they will never lose power in the glass ceiling equilibrium, they are even more likely to admit a candidate of the left type. As c becomes very large ($c > 2$), the right type incumbents in state $i = 3$ stop admitting right type candidates and only admitting left type candidates. In such a case, standard distortion is also the largest: compared with $x^* = \sqrt{2\tau/(3a)}$, candidates of the minority type in a contentious state and candidates of the majority type in a homogenous state are never admitted, and candidates of the majority type in a contentious state and candidates of the minority type in a homogenous state are admitted with a much lower standard than the efficient level.

Dividing all admission probabilities by $2\sqrt{\tau/a}$ and letting $y_i^{b'}$ be the "scaled" admission probabilities, we can illustrate the collegial and glass ceiling equilibria in the following figure:

Insert Figure 1 here.

7.3 Multiple Equilibria and the Influence of Culture

By Propositions 4 and 5, when $c \in (0.345, 0.417)$, both the collegial and glass ceiling equilibria exit. This should not be totally surprising. If the majority right type incumbents in the contentious state $i = 2$ expect that the left type, once in power, will never admit any right type candidate, then they will never admit any left type candidate for the fear of losing control forever. This will lead to the glass ceiling equilibrium. On the other hand, if the majority right type incumbents believe that the left type, once in power, will not be extremely biased, then they will not be extremely biased against left type candidates. This will lead to the collegial equilibrium. Therefore, which equilibrium will result depends on the beliefs of the incumbent members of the club about how each type will behave in the process of admitting new members. If there is a "collegial culture" in the club in the sense that it is commonly believed that politics will not get too much in the way of hiring qualified candidates, the club will indeed be in the collegial equilibrium. Otherwise, if it is commonly believed that the admission policy will be very politicized, then indeed the club will be in the glass ceiling equilibrium.

We should also note that besides the two equilibria discussed above, there is another equilibrium that "links" the two, as can be seen in the Figure 1 above. To link the two equilibria, the admission probabilities (and hence standards) in this third equilibrium have properties that are sometimes opposite of those of the collegial equilibrium (e.g., standard bias decreases rather than increases in c). It is likely that this equilibrium is not stable, but we have not been able to show that.

7.4 Long Run Behavior and Welfare

By Equation (15), the probabilities of the newly admitted member in state $i \in \{0, 1, 2, 3\}$ being the right and left types, respectively, p_i^l are

$$p_i^r = \frac{\bar{v} - v_i^r}{2\bar{v} - v_i^r - v_i^l} = \frac{x_i^r}{x_i^r + x_i^l} \quad p_i^l = \frac{\bar{v} - v_i^l}{2\bar{v} - v_i^r - v_i^l} = \frac{x_i^l}{x_i^r + x_i^l}$$

By Equation (16), the transition probability matrix can be written as

$$\mathbf{P} = (p_{ij})_{i,j\in\{0,1,2,3\}} = \begin{array}{c c} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} p_0^l & p_0^r & 0 & 0 \\ \frac{1}{3}p_1^l & \frac{1}{3}p_1^r + \frac{2}{3}p_1^l & \frac{2}{3}p_1^r & 0 \\ 0 & \frac{2}{3}p_2^l & \frac{2}{3}p_2^r + \frac{1}{3}p_2^l & \frac{1}{3}p_2^r \\ 0 & 0 & p_3^l & p_3^r \end{pmatrix} \end{array}$$

By Equation (17) and symmetry, the stationary probabilities can be found as

$$q_3 = q_0 = \frac{\frac{1}{3}p_2^r}{2\left(\frac{1}{3}p_2^r + p_3^l\right)} \qquad q_2 = q_1 = \frac{p_3^l}{2\left(\frac{1}{3}p_2^r + p_3^l\right)}$$

This is intuitive. By symmetry, $q_3 + q_2 = q_1 + q_0 = 0.5$. Then the ratio of the stationary probability in state 3 over that in state 2, q_3/q_2, should be the ratio of the probability the state moving from state 2 to state 3, $\frac{1}{3}p_2^r$, over the probability the state moving from state 3 to state 2, p_3^l. Note that in the glass ceiling equilibrium, the stationary probabilities are history dependent. If initially the right types are the majority, then $q_3 + q_2 = 1$. But if the club is set up with left type majority, then $q_3 + q_2 = 0$. Assuming equal probability of the two initial conditions, the above formula still applies.

In the first best and the harmonious outcome ($B = c = 0$), since the admission standard is the same for candidates of both types, $p_3^l = p_2^r = 0.5$ hence $q_3 = q_0 = 1/8$ and $q_2 = q_1 = 3/8$. In other words, the club is in the middle states $i = 1, 2$ with probability 0.75 and in the homogenous states $i = 3, 0$ with probability 0.25. When $c > 0$, in the collegial equilibrium, since standard bias is greater in the contentious states than in the homogenous states, $p_2^r > p_3^l$. Furthermore, by Proposition 4, as c increases, the gap between p_2^r and p_3^l must also increase. Therefore, in the collegial equilibrium, q_3 is higher than the efficient level of 1/8 and is increasing in c, where the opposite holds for q_2.

However, when c is large so the club is in the glass ceiling equilibrium, the club will be in the right (resp., left) regime forever if it starts off in the right (resp., left) regime.[14] Suppose it is in the right regime. Then q_3 and q_2 only depend on p_3^l since $p_2^r = 1$. Then as c increases, q_3 decreases but q_2 increases. As $c \geq 2$, $p_3^l = 1$, then $q_3/q_2 = 1/3$, just as in the first best solution or the harmonious equilibrium. This pattern is exhibited in the figure below, where we normalize $q_3 + q_2 = 0.5$ for the glass ceiling equilibrium.

Insert Figure 2 here.

[14]If the club's initial state is in one of the two regimes with equal probabilities, then the expected stationary probability it is in one of the regimes is still 0.5.

Using Equation (20), we can show that the long run welfare of the club is given by

$$U^m = 3Ev + \frac{3}{2}a + \tau - 2\sqrt{a\tau}\gamma^m$$

where $\gamma^m \equiv 4q_3/\left(y_3^r + y_3^l\right) + 4q_2/\left(y_2^r + y_2^l\right)$ and $y_i^{b'} = x_i^{b'}\sqrt{a/\tau}/2$ for $b' = l, r$. Thus it is easy to obtain the following result.

Proposition 6 *(i) In both of the collegial and glass ceiling equilibria, the long run welfare of the club is increasing in a and decreasing in τ and B.*

(ii) In both of the collegial and glass ceiling equilibria, the long run welfare of the club is lower than that in the harmonious equilibrium, and hence is lower than in the first best solution.

(iii) When both equilibria exist, the long run welfare of the club is greater in the collegial equilibrium than in the glass ceiling equilibrium.

Proof: See the Appendix.

The results of Proposition 6 are intuitive. When the spread of the quality distribution a is larger, the benefit from searching is greater and the distortion caused by internal politics is smaller (since c is smaller), thus the long run welfare of the club is greater.[15] When the unit search cost τ is larger, the actual search cost is greater but the distortion caused by internal politics is smaller (again since c is smaller). However, the first direct negative effect dominates the second indirect positive effect, thus the long run welfare of the club decreases in τ. As the total rent increases, politics intensifies and distortion in admission standards increases, reducing the long run welfare of the club.[16]

Note that in both the collegial and glass ceiling equilibria, admission standards are biased relative to those in the harmonious equilibrium in a way such that one type candidate faces a lower standard as another type candidate faces a higher standard. Specifically, in all equilibria, $(y_3^r)^2 + \left(y_3^l\right)^2 = (y_2^r)^2 + \left(y_2^l\right)^2 = 1$. Thus, as the standards of the two type candidates in states $i = 3$ and $i = 2$ diverge from $\hat{y} = \sqrt{2}/2$, the cost term γ^m in the expression of U^m becomes larger (holding q_3 and q_2 fixed). Therefore, the long run welfare in both the collegial and glass ceiling equilibria is lower than the harmonious equilibrium. Since the harmonious equilibrium is inefficient, then the collegial and glass ceiling equilibria are even more inefficient.[17] It also follows

[15] The delay cost is also increasing in a since searching takes more time, but this effect is dominated by the other two positive effects.

[16] If the direct benefit of increasing B is included in the welfare function of the club, then it dominates the negative effect from politics, thus the overall long run welfare of the club will be increasing in B.

[17] Note that in the first best solution the long run welfare of the club has a different expression, that is, the cost term γ is different. While U^m is maximized (γ^m minimized) when $B = c = 0$, it is smaller than U^*.

that the collegial equilibrium yields a greater long run welfare than the glass ceiling equilibrium when both exist.

8 Unanimity Voting in a Three Member Club

Under unanimity voting rule, the club is in the right (resp., left) regime when $i = 3$ (resp., when $i = 0$), and is in the balanced regime when $i = 2$ and $i = 1$. When $i = 3$, Equations (21), (22) and (25) should still hold in the case of unanimity voting.

By Condition (14), the admission criterion in the balanced regime is given by the higher standard between the two types of incumbent members. Abusing notation slightly, we reinterpret v_2^r and v_2^l as the right type incumbents' preferred quality standards in state 2 for right and left type candidates, respectively. With more severe notation abusing, let v_1^r and v_1^l now be the *right* type incumbents' preferred quality standards in state 1 for right and left type candidates, respectively. By symmetry, v_1^r (resp., v_1^l) is the left type incumbent's preferred standard for left (resp., right) type candidate in state 2. Then, the admission criterion in state 2 is $\tilde{v}_2^r = \max\{v_2^r, v_1^l\}$ for right type candidates, and $\tilde{v}_2^l = \max\{v_2^l, v_1^r\}$ for left type candidates.

Proposition 7 *Under unanimity voting rule, in any equilibrium $v_2^r \leq v_1^l$ and $v_2^l \geq v_1^r$. Thus, $\tilde{v}_2^r = v_1^l$ and $\tilde{v}_2^l = v_2^l$.*

Proof: See the Appendix.

Proposition 7 says that under unanimity voting rule, the admission criterion for a candidate is determined by the preferred standard of the incumbent members of his opposite type. Note, however, the preferred standards of the incumbent members are all endogenous and interrelated, and are determined jointly in equilibrium.

Using an approach similar in solving for equilibria under majority rule, we can characterize the equilibria and the associated long run outcomes under unanimity voting rule as follows.

Proposition 8 *Under unanimity voting rule, there are five different kinds of equilibria.*

(i) When $0 < c \equiv B/12\sqrt{a\tau} < 2/3$, a "reverse collegial" equilibrium exists in which candidates of both types are admitted in each state with positive probabilities. However, candidates of the majority type in the balanced regime have lower probability (higher standard) of being admitted than those of the minority type, and are less likely to be admitted as c increases. They are never admitted when c goes to 2/3.

(ii) When $10/29 < c \equiv B/12\sqrt{a\tau} < 2.839$, a "highly political" equilibrium exists in which as in the collegial equilibrium, candidates of both types are admitted in each state with

positive probabilities and candidates of the majority type in the balanced regime have higher probability (lower standard) of being admitted than those of the minority type. However, in the balanced regime politics becomes very intense so that candidates of both types face very high standards and are admitted with very low probabilities, causing long delay in selecting a new member.

(iii) *When* $c > 10/29$*, the glass ceiling equilibrium exists and it is the same as in the majority case. In the long run, the system switches between state 3 and state 2 (resp., state 1 and state 0) if the initial state is 3 or 2 (resp., 1 or 0).*

(iv) *When* $c > 2/3$*, a "minority tyranny" equilibrium exists in which in the balanced regime only candidates of the minority type are admitted. In the long run, the club only switches between state 2 and state 1.*

(v) *When* $c > 2.839$*, an "exclusive" equilibrium exists in which the incumbent members in homogenous states (*$i = 3, 0$*) admit only candidates of their own type. In the long run, the club stays at either state 3 or state 1.*

Proof: See the Appendix.

When each of the equilibria described in Proposition 8 exists can be summarized in the following table:

c	$\left(0, \frac{10}{29}\right)$	$\left(\frac{10}{29}, \frac{2}{3}\right)$	$\left(\frac{2}{3}, 2.839\right)$	(> 2.839)
Reverse Collegial	✓	✓		
Highly Political		✓	✓	
Glass Ceiling		✓	✓	✓
Minority Tyranny			✓	✓
Exclusive				✓

To see the properties of the equilibria more clearly, we illustrate the equilibria under unanimity voting rule in the following figures (again dividing all admission probabilities by $2\sqrt{\tau/a}$). Since the glass ceiling equilibrium has exactly the same property as under majority voting rule, it is omitted here.

Insert Figure 3 here.

Figure 3 above depicts the reverse collegial and minority tyranny equilibria. For $c < 2/3$, in the reverse collegial equilibrium, candidates of the left type are more likely to be admitted than those of the right type in state $i = 2$. This is because the left type incumbent has strong incentives to block right type candidates (as an admission of a right type candidate ensures $i = 2$ conditional on the left type incumbent remaining in the club) so that he may gain the control over the rent allocation. In contrast, a right type incumbent in state $i = 2$ still has 50% chance of being in power after an admission of a left type candidate (conditional on him remaining in the club), thus has lesser incentives to block left type candidates. As c increases, the left type incumbent keeps raising the admission standard for right type candidates in state $i = 2$. When $c \geq 2/3$, he vetoes any right type candidates, the reverse collegial equilibrium becomes extreme, which we call the "minority tyranny" equilibrium. In response to the minority left type incumbent in state $i = 2$, the right type incumbent members also raise the admission standard for left type candidates when c is small ($c < 0.215$), in order not to lose control too quickly to the left type. However, as c increases and the left type incumbent further raises the standard for right type candidates, the right type incumbents yield and lower the admission standard for left type candidates in order to prevent deadlock and incurring large delay cost. In state $i = 3$, the right type incumbents favor left type candidates slightly when c is small (just as in the collegial equilibrium under majority voting), and favor right type candidates slightly when c is in a medium range. When c is large and the minority tyranny equilibrium is played, the right type incumbents in state $i = 3$ increasingly favor left type candidates and admit only left type candidates for $c \geq 6$. This is because in the minority tyranny equilibrium, after admitting a left type candidate in state $i = 3$, the club switches between states $i = 2$ and $i = 1$ forever, which gives a right type incumbent more expected rent than what he can get by staying in state $i = 3$ forever. When B is very large (hence c is large), this effect makes right type incumbents in state $i = 3$ greatly favor left type candidates.

Insert Figure 4 here.

Figure 4 above illustrate the highly political and exclusive equilibria. When c is greater than but close to $10/29$, the highly political equilibrium behaves like the collegial equilibrium under majority voting rule: candidates of both types are admitted in each state with positive probabilities and candidates of the majority type in the balanced regime have higher probability (lower standard) of being admitted than those of the minority type. However, as c increases, candidates of both types face rapidly increasing admission standards hence the club experiences long delay in selecting new members in the balanced regime. The difference with the reverse collegial equilibrium is that the right type incumbents in state $i = 2$ do not want to admit left type candidates when they expect that the left type incumbents in state $i=1$ are not willing to admit right type candidates, and vice versa. Such an expectation is mutually reinforcing, thus leading to low admission probabilities in the two states $i=1,2$ in the balanced regime. In the homogenous states, the incumbent members are reluctant to admit candidates of the opposite type, because of the large political costs and hence low payoffs in the balanced regime. As c increases, the high political equilibrium becomes the exclusive equilibrium, in which incumbent members in the homogenous states only admit candidates of their own type, and in the long run the club stays in one of the homogenous states forever.

Using Equation (20), we can show that the long run welfare under unanimity voting rule is given by

$$U^u = 3Ev + \frac{3}{2}a + \tau - 2\sqrt{a\tau}\gamma^u$$

where $\gamma^u \equiv \frac{4q_3}{y_3^r + y_3^l} + \frac{q_3}{y_1^l + y_2^l}(1 + 3(y_1^l)^2 + 3(y_2^l)^2)$ and $y_i^{b'} = x_i^{b'}\sqrt{a/\tau}/2$ for $b' = l, r$.

Proposition 9 *Under unanimity voting rule,*

(i) In the reverse collegial equilibrium, the long run welfare increases in c when c is relatively small, and achieves its maximum when $c = 0.238$. This maximum welfare is greater than that in the harmonious equilibrium, which is in turn greater than that in the collegial equilibrium under majority voting rule.

(ii) In the highly political equilibrium, the minimum long run welfare is achieved when $c = 2.592$, which is the worst outcome of all the possible equilibria.

(iii) In the glass ceiling equilibrium, the long run welfare is decreasing in c and reaches its minimum for $c \geq 2$.

(iv) In both of the minority tyranny and exclusive equilibria, the long run welfare is always the same as the minimum welfare in the glass ceiling equilibrium (as when $c \geq 2$).

Proof: See the Appendix.

Note that the welfare function in all cases can be expressed as

$$U = 3Ev + \frac{3}{2}a + \tau - 2\sqrt{a\tau}\gamma$$

where $\gamma^* = \sqrt{6}/2$ for the first best; $\hat{\gamma} = \sqrt{2}$ for the harmonious equilibrium; $\gamma^m \equiv 4q_3/\left(y_3^r + y_3^l\right) + 4q_2/\left(y_2^r + y_2^l\right)$ for the majority voting case; and $\gamma^u \equiv 4q_3/(y_3^r + y_3^l) + q_2(1 + 3(y_1^l)^2 + (y_2^l)^2)/(y_1^l + y_2^l)$ for the unanimity voting case. Thus, γ summarizes the total long run expected cost for the club in each case. The smaller γ is, the more efficient it is for the club. The following figure illustrates the result of Propositions 6 and 9 by depicting γ for each of the equilibrium outcome.

Insert Figure 5 here.

From Figure 5, clearly all equilibria outcomes are inefficient compared with the first best solution. As Proposition 9 (i) says, for a significant range of (relatively small) c, the reverse collegial equilibrium under unanimity voting yields greater long run welfare not only than the collegial equilibrium under majority voting, but also than the harmonious equilibrium. The reason is that in the reverse collegial equilibrium, the minority type incumbent in the balanced regime sets a high admission standard for candidates of the majority type in order to gain the control over the rent allocation, and the majority incumbents also set a quite high admission standard for candidates of the minority type in order to maintain the control over the rent allocation when c is not too large (Proposition 8 and Figure 3). Thus politics of this mild form help offset the intertemporal free riding in the harmonious equilibrium, in which incumbent members set admission standards inefficiently low to save on private searching costs. In the homogenous states, admission standards for both types are biased relative to those in the harmonious equilibrium, but the stationary probabilities of the contentious states ($i = 2, 1$) are much higher than those of the homogenous states. Therefore, the long run welfare in the reverse collegial equilibrium under unanimity voting can be greater than that in the harmonious equilibrium. In contrast, in the collegial equilibrium under majority voting, admission standards are biased in all states, leading to lower long run welfare than in the harmonious equilibrium. Therefore, in this range of c, the club's optimal voting rule is unanimity voting, and politics created by unanimity voting rule leads to greater long run welfare than the harmonious outcome.

As can be seen from Figure 5, the long run welfare of the highly political equilibrium under unanimity voting first increases in c and then decreases rapidly in c, and then take a final dip and settles down to a constant. This is because as c increases and politics intensifies, admission standards for both type candidates in the balanced regime quickly increase, reducing the long run welfare. In the end, since the club is getting absorbed into one of the homogenous states, the expected cost in the balanced regime decreases and hence the long run welfare increases. As Proposition 9 (i) says, for a range of $c \in (2, 2.839)$, the long run welfare is lowest in the highly political equilibrium among all other equilibrium outcomes.

Note that in the minority tyranny and exclusive equilibria and the glass ceiling equilibrium for $c \geq 2$, in any stationary long run state candidates of one type are never admitted and those of the other type are admitted with identical probability. This implies that the long run welfare should be the same in all these cases. This is represented by the thick dark line at $\gamma = 2$ in Figure 5.

From Figure 5, for $c \geq 2$, the long run welfare under majority voting in the unique glass ceiling equilibrium is the same as that under unanimity voting rule when either the glass ceiling equilibrium, or the minority tyranny equilibrium, or the exclusive equilibrium for $c > 2.839$ is played. But the majority voting can yield strictly higher long run welfare than unanimity voting if the highly political equilibrium is played when $c \in (2, 2.2839)$. Therefore, in this range of c, the club's optimal voting rule is majority voting.

In the middle range of c, the welfare comparison between majority and unanimity voting rules depends on what equilibrium is played under each of the voting rules.

9 Conclusion

In this paper we build a simple model to study the effects of an organization's internal politics on its hiring of new members. To make the model tractable and as simple as possible, we have made many simplifying assumptions that can be relaxed in future work. We have solved the model when the club size is three and quality distribution is uniform. Though we believe most findings are robust, it is useful to study clubs with larger sizes and with different quality distributions. More importantly, to make welfare comparison simple we have considered distributive politics so that the type profile does not affect welfare directly. In future research, it is interesting to consider the situation in which the type profile directly affects welfare. For example, one can imagine that each member of the controlling type in the club gets a fixed amount of rent (e.g., the club adopts policies that the majority type likes). Or, a candidate brings an additional common value besides his quality *only* to incumbent members of his type (e.g., a new theorist benefits incumbent theorists in a department). In such cases, for a fixed quality profile, the efficient type profile is to have a homogenous club. Alternatively, suppose the production function of the club is modified such that incumbent members of opposite types have synergy. When such synergy is sufficiently strong, diversity will be more efficient than homogeneity. The theoretical framework we develop in this paper can be adapted to study these interesting extensions.

10 Appendix

Proof of Proposition 2: (i) If $\tau \geq n(w - \underline{v})$, then by Equation (2), $\hat{v} = \underline{v}$. Furthermore, by Equation (3), $w = nEv$. Thus, when $\tau \geq n(Ev - \underline{v})$, then $\hat{v} = \underline{v}$ and $w = nEv$ constitute a solution to Equations (3) and (2). As will be clear from (ii), when $\tau \geq n(Ev - \underline{v})$, there is no interior solution, so the corner solution $\hat{v} = \underline{v}$ is unique.

(ii) Suppose $n\hat{v} = w - \tau \geq n\underline{v}$. Plugging $w = n\hat{v} + \tau$ into Equation (3) gives Equation (4). Define the RHS as $G(\hat{v})$. Note that $G(\underline{v}) = n(Ev - \underline{v})$ and $G(\bar{v}) = 0$. Moreover,

$$G'(\hat{v})/n = -1 + F(\hat{v}) < 0$$

Therefore, Equation (4) has a unique solution if $\tau < n(Ev - \underline{v})$, and the solution $\hat{v}$ is decreasing in τ. *Q.E.D.*

Proof of Proposition 4: Substituting (21) and (22) into (25) and simplifying, we can get $4a\tau = (\overline{v} - v_3^r)^2 + (\overline{v} - v_3^l)^2$. Using our variable transformation $x_i^{b'} \equiv (\overline{v} - v_i^{b'})/a$, we have

$$(x_3^r)^2 + \left(x_3^l\right)^2 = 4\tau/a \tag{31}$$

Equations (23) and (24) can be rewritten as

$$\begin{aligned} \frac{5B}{12} + \frac{1}{2}\pi_2^R + \frac{1}{2}\pi_3^R &= \frac{3}{2}\left[\pi_2^R - \tau - v_2^r\right] \\ \frac{B}{4} + \frac{1}{2}\pi_1^R + \frac{1}{2}\pi_2^R &= \frac{3}{2}\left[\pi_2^R - \tau - v_2^l\right] \end{aligned}$$

Substituting these into (26) and simplifying, we can get $4a\tau = (\overline{v} - v_2^r)^2 + (\overline{v} - v_2^l)^2$, or,

$$(x_2^r)^2 + \left(x_2^l\right)^2 = 4\tau/a \tag{32}$$

From Equations (21), (22) and (24), we can get

$$\begin{aligned} \pi_3^R &= \frac{2B}{3} + 3\tau + 3v_3^r \\ \pi_2^R &= \frac{B}{2} + 3\tau + \frac{9}{2}v_3^r - \frac{3}{2}v_3^l \\ \pi_1^R &= \frac{B}{2} + 3\tau + 9v_3^r - 3v_3^l - 3v_2^l \end{aligned}$$

Substituting π_3^R and π_2^R into (23) gives $\frac{B}{6} = 2v_3^r - v_3^l - v_2^r = (\overline{v} - v_2^r) + (\overline{v} - v_3^l) - 2(\overline{v} - v_3^r)$. Thus,

$$x_2^r + x_3^l - 2x_3^r = \frac{B}{6a} \tag{33}$$

Substituting π_1^R, π_2^R into (27) and manipulating terms, we can obtain

$$\begin{aligned}\left[(\overline{v}-v_2^r)+(\overline{v}-v_2^l)\right]B &= 3(\overline{v}-v_2^r)(\overline{v}-v_3^r)+3(\overline{v}-v_2^l)(\overline{v}-v_3^l)\\ &\quad +6(\overline{v}-v_2^l)(\overline{v}-v_2^r)-12(\overline{v}-v_2^l)^2\end{aligned}$$

Then we have

$$(x_2^r+x_2^l)B/a = 3x_2^rx_3^r+3x_2^lx_3^l+6x_2^lx_2^r-12\left(x_2^l\right)^2 \tag{34}$$

Thus, we have four equations (31)-(34) and 4 unknowns: x_3^r, x_3^l, x_2^r and x_2^l. To further simplify things, let $y_i^{b'}=x_i^{b'}\sqrt{a/\tau}/2$, for $i=1,2,3,4$ and $b'=l,r$. Define $c=B/(12\sqrt{a\tau})$. Then (31)-(34) become

$$\begin{aligned}(y_3^r)^2+\left(y_3^l\right)^2 &= 1\\ (y_2^r)^2+\left(y_2^l\right)^2 &= 1\\ y_2^r+y_3^l-2y_3^r &= c\\ y_2^ry_3^r+y_2^ly_3^l+2y_2^ly_2^r-4\left(y_2^l\right)^2 &= 2c(y_2^r+y_2^l)\end{aligned} \tag{35}$$

Substituting the first two equations into the last two gives

$$\begin{aligned}y_2^r+\sqrt{1-(y_3^r)^2}-2y_3^r &= c\\ y_2^ry_3^r+\sqrt{1-(y_2^r)^2}\sqrt{1-(y_3^r)^2}+2y_2^r\sqrt{1-(y_2^r)^2}-4(1-(y_2^r)^2) &= 2c(y_2^r+\sqrt{1-(y_2^r)^2})\end{aligned}$$

Substituting the first equation above into the second gives one equation in terms of y_3^r only, which we can easily solve with numerical methods.

By the first two equations of (35), all $y_i^{b'}$ must be in $(0,1)$. A solution to (35) must also have the following properties:

Claim 1: If $c=0$, then $y_i^{b'}=\sqrt{2}/2$ is a solution, which coincides with the harmonious equilibrium.
Proof: It is easy to check that $y_i^{b'}=\sqrt{2}/2$ is a solution to (35) when $c=0$. Then $x_i^{b'}=2y_i^{b'}\sqrt{\tau/a}=\sqrt{2\tau/a}$. By our calculation in Section 4, in the harmonious equilibrium, $\hat{x}=(\overline{v}-\hat{v})/a=\sqrt{2\tau/a}$. *Q.E.D.*

Claim 2: y_2^l cannot be the largest among the four unknowns. Otherwise, the RHS of the last equation of (35) is negative. Contradiction.

Claim 3: $y_3^r \leq y_3^l$.
Proof: Otherwise, if $y_3^r > y_3^l$, the third equation of (35) implies that

$$y_2^r = 2y_3^r + c - y_3^l > y_3^l$$

Then it must be that $y_2^r > y_3^r > y_3^l > y_2^l$, where the last inequality follows from $(y_3^r)^2 + \left(y_3^l\right)^2 = (y_2^r)^2 + \left(y_2^l\right)^2$. However, substituting the third equation (as the expression of c) into the last equation of (35) gives

$$2\,(y_2^r)^2 + 2y_2^r y_3^l + y_3^l y_2^l - 5y_2^r y_3^r + 4\left(y_2^l\right)^2 - 4y_3^r y_2^l = 0$$

This cannot be consistent with the fact that y_2^r and y_3^r are the largest. Contradiction. *Q.E.D.*

Claim 4: $y_3^r \leq y_2^r$. Otherwise, it must be that $y_2^r < y_3^r \leq y_3^l < y_2^l$, since $(y_3^r)^2 + \left(y_3^l\right)^2 = (y_2^r)^2 + \left(y_2^l\right)^2$. But this violates Claim 2. Contradiction.

Claim 5: $y_2^r \geq y_3^l \geq \frac{\sqrt{2}}{2} \geq y_3^r \geq y_2^l$.
Proof: Suppose $y_3^l > y_2^r$. Then it must be that $y_3^l > \{y_2^r, y_2^l\} \geq y_3^r$. From the third equation of (35), $y_2^r = 2y_3^r + c - y_3^l$. Substituting this into the third term of the LHS of the last equation of (35), we have

$$y_2^r y_3^r - y_2^l y_3^l + 4y_2^l y_3^r - 4\left(y_2^l\right)^2 = 2cy_2^r$$

The LHS is negative when $y_3^l > \{y_2^r, y_2^l\} \geq y_3^r$, because $4y_2^l y_3^r \leq 4\left(y_2^l\right)^2$ and $y_2^r y_3^r < y_2^l y_3^l$. Therefore, it must be that $y_2^r \geq y_3^l$. By Claims 4 and 5 and the fact that $(y_3^r)^2 + \left(y_3^l\right)^2 = (y_2^r)^2 + \left(y_2^l\right)^2 = 1$, it must be that $y_2^r \geq y_3^l \geq \frac{\sqrt{2}}{2} \geq y_3^r \geq y_2^l$. *Q.E.D.*

Using numerical method with Matlab programs, we can verify that a collegial equilibrium exists when $c < 0.417$. Furthermore, it is clear from the numerical solution that y_2^r and y_3^l increase in c, and y_3^r and y_2^l decrease in c when $c < 0.417$. *Q.E.D.*

Proof of Proposition 5: Since Equations (21), (22) and (25) are unchanged from the case of collegial equilibrium, Equation (31) and hence the first equation of (35) should hold in a glass ceiling equilibrium. Moreover, since Equations (21), (22) and (23) are unchanged, Equation (33) and hence the third equation of (35) should also hold in a glass ceiling equilibrium.

Substituting (23) into (29) gives $4a\tau = (\overline{v} - v_2^r)^2$. In other words, $x_2^r = 2\sqrt{\tau/a}$, or, $y_2^r = 1$. Since $v_2^l = \bar{v}$ means $x_2^r = y_2^r = 0$, the second equation of (35) still holds in a glass ceiling equilibrium, just at the corner of $y_2^r = 1$ and $y_2^l = 0$.

From the first and third equations of (35), $(y_3^r)^2 + \left(y_3^l\right)^2 = 1$ and $y_3^l - 2y_3^r = c - 1$, we can obtain the following solution:

$$
\begin{aligned}
y_3^r &= \frac{1}{5}\left(\sqrt{4+2c-c^2}-2c+2\right)\\
y_3^l &= \frac{1}{5}\left(2\sqrt{4+2c-c^2}+c-1\right)
\end{aligned}
$$

Then from Equations (29) and (30), we can get

$$
\begin{aligned}
\pi_3^R &= \frac{2B}{3}+3\tau+3v_3^r\\
\pi_2^R &= 3\overline{v}+3\tau+\frac{3}{4}B-3\sqrt{a\tau}\left(1+y_3^r\right)\\
\pi_1^R &= 3\overline{v}+3\tau-6\sqrt{a\tau}
\end{aligned}
$$

Substituting π_2^R and π_1^R into (28), we get $y_3^r<2c$. This is satisfied if and only if $c>\frac{10}{29}$. It can be checked that when $c>\frac{10}{29}$, $y_3^l>y_3^r$. Furthermore, y_3^l is increasing in c and y_3^r is decreasing in c. Also notice that when $c>2$, $\frac{1}{5}\left(\sqrt{4+2c-c^2}-2c+2\right)<0$. Actually in this case it's not difficult to verify that in the glass ceiling equilibrium, $y_3^r=0, y_3^l=1$. *Q.E.D.*

Proof of Proposition 6: Using Equation (20), we have

$$
\begin{aligned}
U &= 6\left[q_3\left(p_3^r\frac{\overline{v}+v_3^r}{2}+p_3^l\frac{\overline{v}+v_3^l}{2}\right)+q_2\left(p_2^r\frac{\overline{v}+v_2^r}{2}+p_2^l\frac{\overline{v}+v_2^l}{2}\right)\right]-2\tau\left[q_3\frac{1-x_3}{x_3}+q_2\frac{1-x_2}{x_2}\right]\\
&= 3\overline{v}-6\left[q_3\left(p_3^r\frac{\overline{v}-v_3^r}{2}+p_3^l\frac{\overline{v}-v_3^l}{2}\right)+q_2\left(p_2^r\frac{\overline{v}-v_2^r}{2}+p_2^l\frac{\overline{v}-v_2^l}{2}\right)\right]-2\tau\left[q_3\frac{1-x_3}{x_3}+q_2\frac{1-x_2}{x_2}\right]\\
&= 3\overline{v}-3a\left[q_3\left(p_3^r x_3^r+p_3^l x_3^l\right)+q_2\left(p_2^r x_2^r+p_2^l x_2^l\right)\right]-2\tau\left[q_3\frac{1-\frac{x_3^r+x_3^l}{2}}{\frac{x_3^r+x_3^l}{2}}+q_2\frac{1-\frac{x_2^r+x_2^l}{2}}{\frac{x_2^r+x_2^l}{2}}\right]\\
&= 3\overline{v}-6\sqrt{a\tau}\left[q_3\left(p_3^r y_3^r+p_3^l y_3^l\right)+q_2\left(p_2^r y_2^r+p_2^l y_2^l\right)\right]-2\tau\left[q_3\frac{\sqrt{\frac{a}{\tau}}-\left(y_3^r+y_3^l\right)}{y_3^r+y_3^l}+q_2\frac{\sqrt{\frac{a}{\tau}}-\left(y_2^r+y_2^l\right)}{y_2^r+y_2^l}\right]\\
&= 3\overline{v}-2\sqrt{a\tau}\left[q_3\left(\frac{1}{y_3^r+y_3^l}+3\left(p_3^r y_3^r+p_3^l y_3^l\right)\right)+q_2\left(\frac{1}{y_2^r+y_2^l}+3\left(p_2^r y_2^r+p_2^l y_2^l\right)\right)\right]+2\tau\left(q_3+q_2\right)\\
&= 3\overline{v}-2\sqrt{a\tau}\left[q_3\left(\frac{1}{y_3^r+y_3^l}+3\frac{\left(y_3^r\right)^2+\left(y_3^l\right)^2}{y_3^r+y_3^l}\right)+q_2\left(\frac{1}{y_2^r+y_2^l}+3\frac{\left(y_2^r\right)^2+\left(y_2^l\right)^2}{y_2^r+y_2^l}\right)\right]+\tau\\
&= 3\overline{v}-2\sqrt{a\tau}\left[\frac{4q_3}{y_3^r+y_3^l}+\frac{4q_2}{y_2^r+y_2^l}\right]+\tau\\
&\equiv 3Ev+\frac{3}{2}a+\tau-2\sqrt{a\tau}\gamma^m
\end{aligned}
$$

where the second last equation follows Equation (35), and $\gamma^m \equiv 4q_3/\left(y_3^r + y_3^l\right) + 4q_2/\left(y_2^r + y_2^l\right)$.

Thus,

$$\frac{\partial U}{\partial a} = \frac{3}{2} - \sqrt{\frac{\tau}{a}}\gamma^m - 2\sqrt{a\tau}\frac{\partial \gamma^m}{\partial c}\frac{\partial c}{\partial a} = \frac{3}{2} - \frac{1}{2}\sqrt{\frac{4\tau}{a}}\left(\gamma^m - \frac{\partial \gamma^m}{\partial c}c\right)$$

Using numerical solutions we can show for both the collegial and glass ceiling equilibria,

$$\frac{3}{2} - \frac{1}{2}\left(\gamma^m - \frac{\partial \gamma^m}{\partial c}c\right) > 0$$

and $\gamma^m - \frac{\partial \gamma^m}{\partial c}c > 0$. Since $a > 4\tau$, we have $\partial U/\partial a > 0$ for both equilibria.

For the comparative statics with respect to τ, note that

$$\frac{\partial U}{\partial \tau} = 1 - \sqrt{\frac{a}{\tau}}\gamma^m - 2\sqrt{a\tau}\frac{\partial \gamma^m}{\partial c}\frac{\partial c}{\partial \tau} = 1 - 2\sqrt{\frac{a}{4\tau}}\left(\gamma^m - \frac{\partial \gamma^m}{\partial c}c\right)$$

By numerical solutions, we have for both the collegial and glass ceiling equilibria,

$$1 - 2\left(\gamma^m - \frac{\partial \gamma^m}{\partial c}c\right) < 0$$

Since $a > 4\tau$, we have $\partial U/\partial \tau < 0$.

For the comparative statics with respect to B, note that

$$\frac{\partial U}{\partial B} = -2\sqrt{a\tau}\frac{\partial \gamma^m}{\partial c}\frac{\partial c}{\partial B} = -\frac{1}{6}\frac{\partial \gamma^m}{\partial c}$$

Since $\frac{\partial \gamma^m}{\partial c} > 0$, we have that $\partial U/\partial B < 0$.

Since in the harmonious equilibrium,

$$U^h = 3Ev + \frac{3}{2}a + \tau - 2\sqrt{2a\tau}$$

and by numerical solutions $\gamma^m > \sqrt{2}$, for any collegial or glass ceiling equilibrium we know that the long run welfare under the majority voting rule is always lower than that in the harmonious equilibrium.

When both the collegial and glass ceiling equilibria exist, we can compare

$$U^C - U^G = 2\sqrt{a\tau}\left(\gamma^G - \gamma^C\right)$$

where the superscripts C and G represent the collegial and glass ceiling equilibria, respectively. By numerical solutions, we have $\gamma^G - \gamma^C > 0$. Therefore, $U^C > U^G$. *Q.E.D.*

Proof of Proposition 7: First we can show the following result:

Lemma 1

$$\begin{aligned}(x_2^r)^2 + \left(x_2^l\right)^2 &= \frac{4\tau}{a} + \left[\max\{x_2^r, x_1^l\} - x_1^l\right]^2 + \left[\max\{x_2^l, x_1^r\} - x_1^r\right]^2 \\ (x_1^r)^2 + \left(x_1^l\right)^2 &= \frac{4\tau}{a} + \left[\max\{x_2^r, x_1^l\} - x_2^r\right]^2 + \left[\max\{x_2^l, x_1^r\} - x_2^l\right]^2\end{aligned}$$

Proof: Since the admission criterion is now given by $\tilde{v}_2^r = \max\{v_2^r, v_1^l\}$ and $\tilde{v}_2^l = \max\{v_2^l, v_1^r\}$, Equations (26) should be modified as follows:

$$\begin{aligned}\pi_2^R &= \frac{\overline{v} - \tilde{v}_2^r}{3a}\left[\frac{5B}{12} + \frac{1}{2}\pi_2^R + \frac{1}{2}\pi_3^R\right] + \frac{\left[\overline{v}^2 - (\tilde{v}_2^r)^2\right]}{4a} + \frac{\tilde{v}_2^r - \underline{v}}{2a}[\pi_2^R - \tau] \\ &\quad + \frac{\overline{v} - \tilde{v}_2^l}{3a}\left[\frac{B}{4} + \frac{1}{2}\pi_1^R + \frac{1}{2}\pi_2^R\right] + \frac{\left[\overline{v}^2 - \left(\tilde{v}_2^l\right)^2\right]}{4a} + \frac{\tilde{v}_2^l - \underline{v}}{2a}[\pi_2^R - \tau]\end{aligned}$$

When $v_2^r < \overline{v}$, $v_2^l < \overline{v}$, Equations (23) and (24) are still valid. So we can rewrite the above equation as

$$\begin{aligned}\pi_2^R &= \frac{\overline{v} - v_2^r}{2a}\left[\pi_2^R - \tau - v_2^r\right] + \frac{\left[\overline{v}^2 - (v_2^r)^2\right]}{4a} + \frac{v_2^r - \underline{v}}{2a}[\pi_2^R - \tau] \\ &\quad + \frac{\overline{v} - v_2^l}{2a}\left[\pi_2^R - \tau - v_2^l\right] + \frac{\left[\overline{v}^2 - \left(v_2^l\right)^2\right]}{4a} + \frac{v_2^l - \underline{v}}{2a}[\pi_2^R - \tau] \\ &\quad + \frac{v_2^r - \tilde{v}_2^r}{2a}\left[\pi_2^R - \tau - v_2^r\right] + \frac{\left[(v_2^r)^2 - (\tilde{v}_2^r)^2\right]}{4a} + \frac{\tilde{v}_2^r - v_2^r}{2a}[\pi_2^R - \tau] \\ &\quad + \frac{v_2^l - \tilde{v}_2^l}{2a}\left[\pi_2^R - \tau - v_2^r\right] + \frac{\left[\left(v_2^l\right)^2 - \left(\tilde{v}_2^l\right)^2\right]}{4a} + \frac{\tilde{v}_2^l - v_2^l}{2a}[\pi_2^R - \tau]\end{aligned}$$

Notice when $v_2^r = \overline{v}$ $\left(\text{resp., } v_2^l = \overline{v}\right)$, although Equation (23) (resp., (24)) does not hold, this expression for π_2^R is still correct since $\overline{v} = v_2^r = \tilde{v}_2^r$ $\left(\text{resp., } \overline{v} = v_2^l = \tilde{v}_2^l\right)$.

Simplifying terms and multiplying both sides by $4a$ we can easily get

$$(\overline{v} - v_2^r)^2 + \left(\overline{v} - v_2^l\right)^2 = 4a\tau + (\tilde{v}_2^r - v_2^r)^2 + \left(\tilde{v}_2^l - v_2^l\right)^2$$

Noting that

$$\frac{\tilde{v}_2^r - v_2^r}{a} = \frac{\bar{v} - v_2^r}{a} - \frac{\bar{v} - \tilde{v}_2^r}{a} = x_2^r - \min\{x_2^r, x_1^l\} = \max\{x_2^r, x_1^l\} - x_1^l$$

and similarly,

$$\frac{v_1^l - v_2^l}{a} = \max\{x_2^l, x_1^r\} - x_1^r$$

we get the first statement of the lemma. Using the same method on Equation (27), and with the fact that when $v_1^l < \overline{v}$, $v_1^r < \overline{v}$

$$\begin{aligned} \frac{2}{3}\left[\frac{3}{2}v_1^r + \frac{B}{2} + \pi_2^R\right] &= \pi_1^R - \tau && (36) \\ \frac{2}{3}\left[\frac{3}{2}v_1^l + \pi_1^R\right] &= \pi_1^R - \tau && (37) \end{aligned}$$

we can prove the second statement of the lemma. *Q.E.D.*

To prove the proposition, let's first assume that all the quality standards are less than $\overline{v}$. From Equations (21)-(24) and (36)-(37), we can eliminate all the πs to get

$$\begin{aligned} x_2^r + x_3^l - 2x_3^r &= \frac{B}{6a} && (38) \\ x_3^r - x_2^l + x_2^r - x_1^l &= \frac{B}{3a} && (39) \\ x_2^l + x_1^r - 2x_1^l &= \frac{B}{2a} && (40) \end{aligned}$$

We now eliminate all the other possibilities to prove the proposition.

(a) Suppose $v_1^l \geq v_2^r, v_1^r > v_2^l$, then $x_1^l \leq x_2^r, x_1^r < x_2^l$. By the above lemma, we have

$$\begin{aligned} (x_2^r)^2 + \left(x_2^l\right)^2 &= \frac{4\tau}{a} + \left(x_2^r - x_1^l\right)^2 + \left(x_2^l - x_1^r\right)^2 \\ (x_1^r)^2 + \left(x_1^l\right)^2 &= \frac{4\tau}{a} \end{aligned}$$

Substituting the second equation into the first equation, we can get $x_2^l x_1^r + x_2^r x_1^l = (x_1^r)^2 + \left(x_1^l\right)^2$. But this cannot hold, because by $x_1^l \leq x_2^r$ and $x_1^r < x_2^l$, the RHS is less than the LHS.

(b) Suppose $v_1^l < v_2^r, v_1^r \leq v_2^l$, then $x_1^l > x_2^r, x_1^r \geq x_2^l$. Following the same method as in part (a), we can get $x_2^r x_1^l + x_2^l x_1^r = (x_2^r)^2 + \left(x_2^l\right)^2$, which is impossible since $x_1^l > x_2^r, x_1^r \geq x_2^l$.

(c) Suppose $v_1^l < v_2^r, v_1^r > v_2^l$, then $x_1^l > x_2^r, x_1^r < x_2^l$. Equation (40) and $x_1^r < x_2^l$ imply that $x_1^l < x_2^l$. Equation (39) and $x_1^l > x_2^r$ imply that $x_2^l < x_3^r$. Thus, we have $x_2^r < x_1^l < x_2^l < x_3^r$. By Equation (38), we must have $x_3^l > x_3^r$. From Lemma 1 we have

$$\begin{aligned} (x_2^r)^2 + \left(x_2^l\right)^2 &= \frac{4\tau}{a} + \left(x_2^l - x_1^r\right)^2 \\ (x_1^r)^2 + \left(x_1^l\right)^2 &= \frac{4\tau}{a} + \left(x_1^l - x_2^r\right)^2 \end{aligned}$$

Summing them up and substituting $\frac{4\tau}{a}$ by $(x_3^r)^2 + \left(x_3^l\right)^2$ (since Equation (31) is still valid), we can get

$$(x_3^r)^2 + \left(x_3^l\right)^2 = x_1^r x_2^l + x_2^r x_1^l \tag{41}$$

But this contradicts the fact that x_3^l and x_3^r are greater than all the four variables on the RHS.

In summary, in an interior equilibrium under unanimity voting it must be that $v_1^l \geq v_2^r$ and $v_1^r \leq v_2^l$.

Now consider some of the standards are greater than $\overline{v}$. Part (a) and (b) of the above proof are still valid. For part (c), assume $\widehat{v}_3^r$ satisfies Equation (21), which means

$$\frac{2}{3}\left[\frac{3}{2}\widehat{v}_3^r + \frac{B}{3} + \pi_3^R\right] = \pi_3^R - \tau$$

and do the same thing to Equation $(22)-(24), (36)-(37)$, we can get $\widehat{v}_3^l, \widehat{v}_2^r, \widehat{v}_2^l, \widehat{v}_1^r, \widehat{v}_1^l$ respectively. It's obvious that $v_i^{b'} = \min\left\{\widehat{v}_i^{b'}, \overline{v}\right\}$.

Define $\widehat{x}_i^{b'} \equiv \frac{\overline{v}-\widehat{v}_i^{b'}}{a}$, then $x_i^{b'} = \max\left\{\widehat{x}_i^{b'}, 0\right\}$ and $(38) - (40)$ become

$$\begin{aligned} \widehat{x}_2^r + \widehat{x}_3^l - 2\widehat{x}_3^r &= \frac{B}{6a} && (42)\\ \widehat{x}_3^r - \widehat{x}_2^l + \widehat{x}_2^r - \widehat{x}_1^l &= \frac{B}{3a} && (43)\\ \widehat{x}_2^l + \widehat{x}_1^r - 2\widehat{x}_1^l &= \frac{B}{2a} && (44) \end{aligned}$$

Since $x_1^l > x_2^r, x_1^r < x_2^l$, it's straightforward that $\widehat{x}_1^l > \widehat{x}_2^r, \widehat{x}_1^r < \widehat{x}_2^l$. So we can follow the same analysis as in part (c) above to get that $\widehat{x}_3^r$ and $\widehat{x}_3^l$ are greater than the other four $\widehat{x}_i^{b'}$. Noting that at least one $\widehat{x}_2^{b'}$ should be positive (otherwise in state 2 the club will not hire any candidate and get negative infinite expected utility), So $\widehat{x}_3^r$ and $\widehat{x}_3^l$ must be positive. Then we have $x_3^r = \widehat{x}_3^r, x_3^l = \widehat{x}_3^l$ and $\max\left\{x_1^r, x_2^l, x_2^r, x_1^l\right\} = \max\left\{\widehat{x}_1^r, \widehat{x}_2^l, \widehat{x}_2^r, \widehat{x}_1^l\right\} < \min\left\{\widehat{x}_3^r, \widehat{x}_3^l\right\} = \min\left\{x_3^r, x_3^l\right\}$. Also notice that Equation (41) is always valid whether the standard is greater than $\overline{v}$ or not. So we can get the same contradiction as in part (c) above. *Q.E.D.*

Proof of Proposition 8: Use lemma 1 and proposition 7 we can easily get the following results:

$$\begin{aligned} (x_2^r)^2 + \left(x_2^l\right)^2 &= \frac{4\tau}{a} + \left[x_2^r - x_1^l\right]^2 && (45)\\ (x_1^r)^2 + \left(x_1^l\right)^2 &= \frac{4\tau}{a} + \left[x_1^r - x_2^l\right]^2 && (46) \end{aligned}$$

So for solutions with quality standards lower than $\overline{v}$, we have six equations $(31), (38) - (40), (45) - (46)$, and six unknowns $x_3^r, x_3^l, x_2^r, x_2^l, x_1^r, x_1^l$. Let

$$y_i^j = \sqrt{\frac{a}{4\tau}}x_i^j, \quad c = \frac{B}{12\sqrt{a\tau}} \tag{47}$$

Then we can solve the system by Matlab.

Part (i) and (ii) come directly from the numerical solution.

(iii) Assume $\tilde{v}_2^l = v_2^l = \overline{v}$, then the following inequalities must be satisfied

$$\frac{2}{3}\left[\frac{3}{2}\overline{v} + \frac{B}{4} + \frac{1}{2}\pi_1^R + \frac{1}{2}\pi_2^R\right] \leq \pi_2^R - \tau \tag{48}$$

By the fact that $x_2^l = 0$ and Equations $(45), (46)$, we can easily derive

$$x_2^r = x_1^l = \sqrt{\frac{4\tau}{a}}$$

Now the case is almost the same as the glass ceiling equilibrium in the majority voting case. We have

$$\begin{aligned}
\pi_3^R &= 3\overline{v} + 3\tau + \frac{2B}{3} - 6\sqrt{a\tau}y_3^r \\
\pi_2^R &= 3\overline{v} + 3\tau + \frac{3}{4}B - 3\sqrt{a\tau} - 3\sqrt{a\tau}y_3^r \\
\pi_1^R &= 3\overline{v} + 3\tau - 6\sqrt{a\tau}
\end{aligned}$$

in which

$$y_3^r = \frac{1}{5}\left(\sqrt{4 + 2c - c^2} - 2c + 2\right)$$

Substituting π_1^R and π_2^R into (48), we can get $c > \frac{10}{29}$. In the long run, since $\widetilde{x}_2^l = 0$, we know $p_2^l = p_1^r = 0$. So $p_{21} = p_{12} = 0$. *Q.E.D.*

(iv) Assume $\tilde{v}_2^r = v_1^l = \overline{v}$, we must have

$$\frac{2}{3}\left[\frac{3}{2}\overline{v} + \pi_1^R\right] \leq \pi_1^R - \tau \tag{49}$$

Also as in the previous part we can prove that

$$x_2^l = x_1^r = \sqrt{\frac{4\tau}{a}}$$

So

$$v_2^l = v_1^r = \overline{v} - 2\sqrt{a\tau}$$

Using

$$\begin{aligned}
\frac{2}{3}\left[\frac{3}{2}v_2^l + \frac{B}{4} + \frac{1}{2}\pi_1^R + \frac{1}{2}\pi_2^R\right] &= \pi_2^R - \tau \\
\frac{2}{3}\left[\frac{3}{2}v_1^r + \frac{B}{2} + \pi_2^R\right] &= \pi_1^R - \tau
\end{aligned}$$

we can get

$$\pi_1^R = \frac{3}{4}B + 3\overline{v} + 3\tau - 6\sqrt{a\tau}$$

Substituting π_1^R into (49), we have $c \geq \frac{2}{3}$. In the long run, since $\widetilde{x}_2^r = 0$, we know $p_2^r = p_1^l = 0$. So $p_{23} = p_{10} = 0$ and $q_3 = q_0 = 0$. *Q.E.D.*

(v) Assume that $v_3^l = \overline{v}$. Then we must have

$$\frac{2}{3}\left[\frac{3}{2}\overline{v} + \frac{B}{2} + \pi_2^R\right] \leq \pi_3^R - \tau \tag{50}$$

By Equation (31) we have $v_3^r = \overline{v} - 2\sqrt{a\tau}$. So by (21) we can get $\pi_3^R = \frac{2}{3}B + 3\overline{v} + 3\tau - 6\sqrt{a\tau}$. Substituting it into Equation (23), we have $\pi_2^R = \frac{3}{4}B + 3\overline{v} + 3\tau - 3\sqrt{a\tau} - 3\sqrt{a\tau}y_2^r$. Using the expressions of π_3^R and π_2^R, we can simplify (50) to $y_2^r \geq 2 + c$. By the same method we used in the proof of Proposition 4, we can find four equations here:

$$\begin{aligned}
1 - y_2^l + y_2^r - y_1^l &= 2c \\
y_2^l + y_1^r - 2y_1^l &= 3c \\
(y_2^r)^2 + \left(y_2^l\right)^2 &= 1 + \left(y_2^r - y_1^l\right)^2 \\
(y_1^r)^2 + \left(y_1^l\right)^2 &= 1 + \left(y_1^r - y_2^l\right)^2
\end{aligned}$$

Use numerical method we find that when $c \geq 2.839$, this kind of solution exists. Also since in the long run $x_3^l = 0$, $p_3^l = p_0^1 = 0$. So $p_{32} = p_{01} = 0$ and $q_2 = q_1^r = 0$. *Q.E.D.*

All we need to do now is try to check that there is no equilibrium where $v_3^r = \overline{v}$ and all the other quality standards smaller than $\overline{v}$. By the same method as in part (v), we can use matlab program to verify this result. *Q.E.D.*

Proof of Proposition 9: Using Equation (20) and the facts $\widetilde{v}_2^r = v_1^l, \widetilde{v}_2^l = v_2^l$, we can show

$$\begin{aligned}
U &= 3\overline{v} - 2\sqrt{a\tau}\left[q_3\left(\frac{4}{y_3^r + y_3^l}\right) + q_2\left(\frac{1}{y_1^l + y_2^l} + 3\frac{\left(y_1^l\right)^2 + \left(y_2^l\right)^2}{y_1^l + y_2^l}\right)\right] + \tau \\
&\equiv 3Ev + \frac{3}{2}a + \tau - 2\sqrt{a\tau}\gamma^u
\end{aligned}$$

where

$$\gamma^u \equiv q_3\left(\frac{4}{y_3^r + y_3^l}\right) + q_2\left(\frac{1}{y_1^l + y_2^l} + 3\frac{\left(y_1^l\right)^2 + \left(y_2^l\right)^2}{y_1^l + y_2^l}\right)$$

From numerical solutions we can get the first two parts of the proposition. The first statement of part (iii) comes from the same proof as in Proposition 6.

When $c \geq 2$, in the glass ceiling equilibrium $v_3^r = \overline{v} \Rightarrow y_3^r = 0 \Rightarrow y_3^l = 1$. Also in this kind of equilibrium $y_1^l = y_2^r = 1$ and $y_2^l = 0$. So $\gamma^u = q_3 * 4 + q_2 * 4 = 2$ is a constant, which implies the long run welfare keeps the same when $c \geq 2$. Since γ^u is increasing in c when $c \leq 2$, we know that when $c \geq 2$ the welfare is minimum.The second statement of part (iii) is proved.

Notice that in the minority tyranny equilibrium, $q_3 = 0, q_2 = \frac{1}{2}$ and $y_1^l = 0, y_2^l = 1$, so $\gamma^u = \frac{1}{2} * 4 = 2$. In the exclusive equilibrium, $q_3 = \frac{1}{2}, q_2 = 0$ and $y_3^l = 0, y_3^r = 1$, so $\gamma^u = \frac{1}{2} * 4 = 2$. Then part (iv) is finished. *Q.E.D.*

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Figure 1: Majority Voting Admission Probabilities ($\times\sqrt{\frac{a}{4\tau}}$)

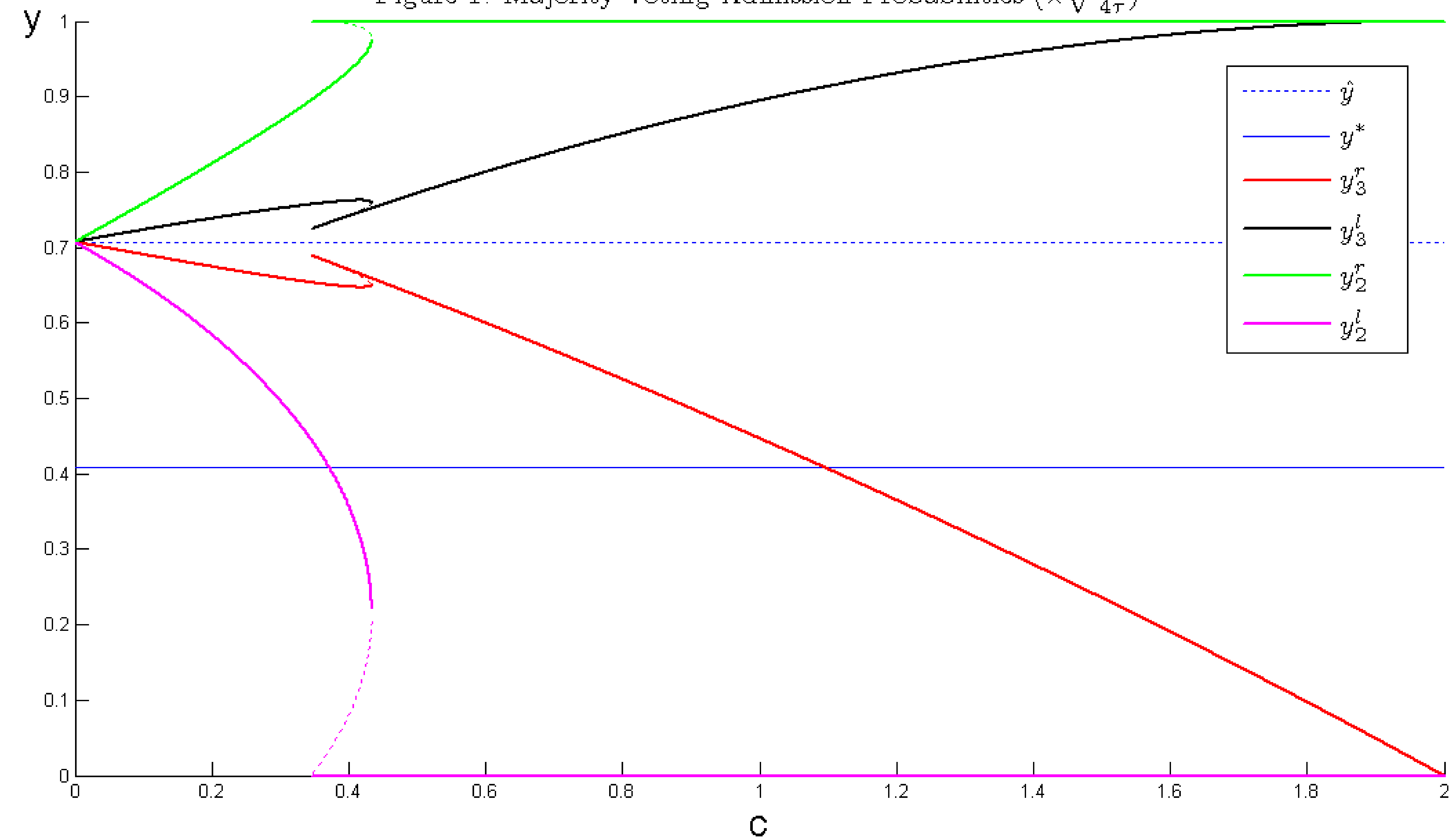

Figure 2: Majority Voting Stationary Probabilities

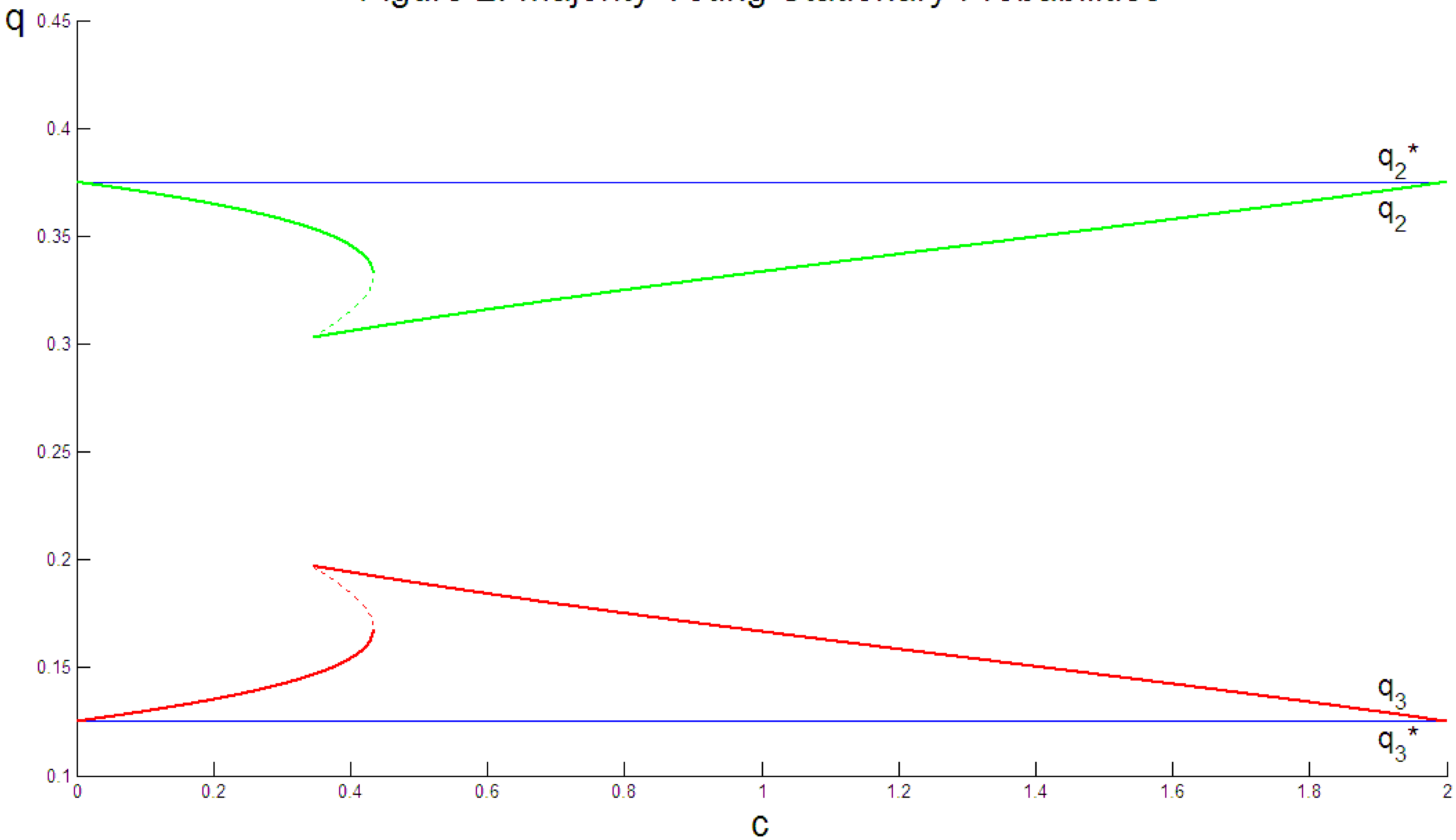

Figure 3: Unanimity Voting Admission Probabilities ($\times\sqrt{\frac{a}{4\tau}}$) –Reverse Collegial and Minority Tyranny

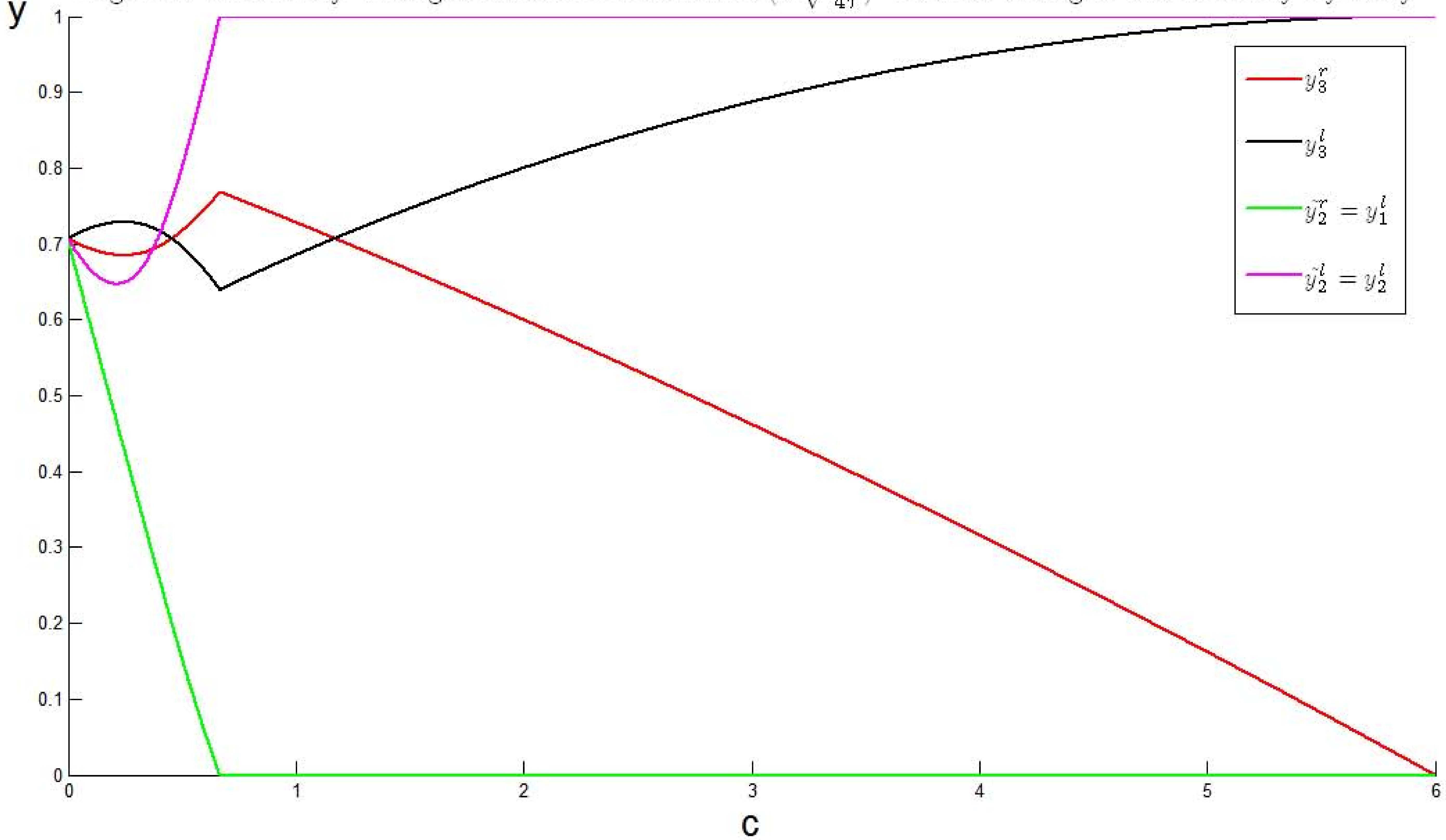

Figure 4: Unanimity Voting Admission Probabilities ($\times\sqrt{\frac{a}{4\tau}}$) –Highly Political and Exclusive

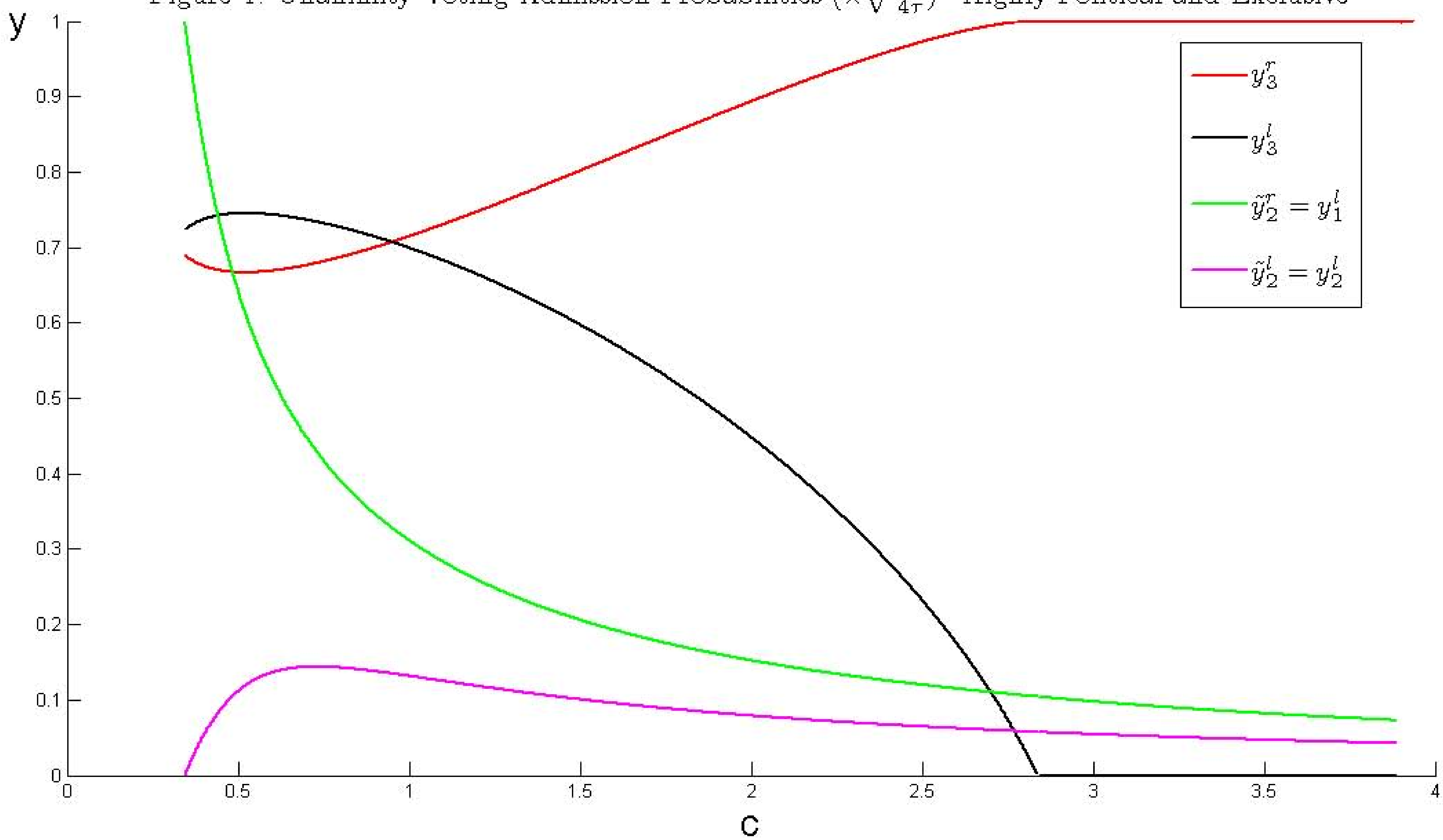

Figure 5: Long Run Welfare Comparison